Math 275: Introduction to Non-Linear Algebra

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Homework # 2, due Monday, February 3

1. Let α and β be the unique positive real roots of the polynomials $f(x) = x^3 + 3x^2 + 3x - 1$ and $g(x) = 8x^4 - 16x^3 + 12x^2 - 4x - 3$ respectively.
   - Express α and β in radicals, and in floating point approximations.
   - Compute the minimal polynomials of $\alpha + \beta, \alpha - \beta, \alpha \cdot \beta$, and α/β.
2. Explain the difference between the lexicographic term order and the reverse lexicographic term order. How are they applied? What do they mean geometrically? Illustrate your points with some examples.
3. Determine the ideal of polynomial relations among the ...
   - ... ten 2×2-minors of a 2×5-matrix;
   - ... seven principal minors of a symmetric 3×3-matrix;
   - ... ten off-diagonal entries of a symmetric 5×5-matrix of rank 2.
4. Determine the prime ideal of polynomial relations among the traces of the eight matrix words $A_i A_j A_k$ where A_0 and A_1 are 2×2-matrices. How is this question related to the study of *Hidden Markov Models*?
5. *[CBMS 4.6]* Use Sylvester's formula for $\mathrm{Res}_{3,3,3}$ to solve the equations
$$\begin{aligned} (x+y-z)(x+2y-3z)(x+4y-9z) &= \alpha \\ (x-y+z)(x-2y+3z)(x-4y+9z) &= \beta \\ (-x+y+z)(-x+2y+3z)(-x+4y+9z) &= \gamma \end{aligned}$$
   where α, β, γ are parameters. How does x depend on these parameters? Show that there is a unique real solution for $\alpha = 13, \beta = 17, \gamma = 19$.
6. *[CBMS 4.7]* Give an exact formula for the resultant of three bilinear equations in (x, y). Now do the same for three biquadratic equations.