Math 113, **Final Exam**
Thursday, August 12, 10:00–12:00am

This exam is open book. You may use the text book (to which you can refer) and your own notes, but no electronic devices. Please write your answers in a blue note book. There are seven problems, the first six are worth 7 points and the last one is worth 8 points, for a total of 50 points. Answers without justification will not receive credit.

(1) Consider the field $K = \mathbf{GF}(25)$ with 25 elements. Classify both the additive group $(K, +)$ and the multiplicative group $(K^*, \cdot)$ according to the Fundamental Theorem of Finitely Generated Abelian Groups.

(2) Fix the ring $R = \mathbf{Z}_6$ and its multiplicatively closed subset $T = \{1, 5\}$. How many elements are there in the partial ring of quotients $Q(R, T)$?

(3) The four edges of a square of cardboard are painted with n colors. The same color can be used on any number of edges. Apply Burnside's Theorem to find a formula for the number of distinguishable colorings.

(4) Find all ring homomorphisms $\mathbf{Z} \times \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$. How many are there?

(5) Find a composition series of the group $S_3 \times S_3$. Is $S_3 \times S_3$ solvable?

(6) Show that the symmetric group S_4 can be generated by two elements.

(7) Prove that there is no simple group of order 96.