Math 113 (Bernd Sturmfels), **Midterm Exam # 2**

Tuesday, August 1, 9:00 a.m.–10:00 a.m.

Please start by writing your name and your student ID on the cover of your blue book. **This exam is closed book**. Do not use any notes, calculators, cell phones etc. Show all your work, and write full sentences if time permits. Each problem is worth 20 points, for a total of 100 points.

(1) Classify all groups of order 65 up to isomorphism.

(2) Show that the alternating group A_4 is generated by the two simple 3-cycles (123) and (234).

(3) The ring of integers $\mathbf{Z}$ is a principal ideal domain, so each ideal in $\mathbf{Z}$ is generated by a single element. Consider the ideals

$$I = \langle 12, 20 \rangle \text{ and } J = \langle 18, 30 \rangle.$$

Find a single generator for each of I, J, $I + J$, $I \cap J$, and $I \cdot J$.

(4) Can you find examples of ideals I in commutative rings R with the following properties ?

(a) I is principal and maximal,

(b) I is principal but not prime,

(c) I is maximal but not prime,

(d) I is maximal but not principal,

(e) I is principal and prime but not maximal.

(5) Consider the two polynomials $f = x^4 + x^3 + 7$ and $g = x^2 - 2$.

(a) Compute the remainder of f divided by g.

(b) Compute the remainder of g divided by the result of part (a).

(c) Determine the ideal in $\mathbf{Q}[x]$ generated by f and g.