WORKSHOP IN COMPUTABILITY THEORY
PARIS - JULY 2010

DENIS HIRSCHFELDT AND ANTONIO MONTALBÁN

Second session of the Workshop on Computability Theory 2010.
Satellite to the Logic Colloquium 2010.

Location

University of Chicago Center in Paris.
6, rue Thomas Mann, 75013 Paris, France
Right across the street from the location of the Logic Colloquium 2010.

Web page: http://www.math.uchicago.edu/~antonio/WCTParis.html

Schedule

	Friday 23rd	Saturday 24th
9:20 - 10:00am	coffee at 10	Antonin Kučera
10:20 - 11:00am	Richard A. Shore	David Diamondstone
11:20 - 12:00pm	Alberto Marcone	Wolfang Merkle
Lunch break		
2:40 - 3:20pm	Alexandra Soskova	Mariya Soskova
3:40 - 4:20pm	Alexey Stukachev	Ekaterina Fokina
4:40 - 5:20pm	Valentina Harizanov	Barbara Csima

Titles and Abstracts

Richard A. Shore.
Title: The n-r.e. degrees: undecidability and Σ_1−substructures (joint work with Cai and Slaman)
Abstract: We prove that the n-r.e. degrees, D_n, are undecidable for every $n \geq 2$ and that D_n is not a Σ_1 elementary substructure of D_m for any $2 \leq n < m$. (The cases for $n = 1$ are already known: Harrington and Shelah (1982) and Yang and Yu (2006).) The specific technical theorems we prove are as follows:

Theorem: Given a recursive partial order $(\omega, \leq_*)$, there exist n-r.e. sets G_i for each $i \in \omega$, an n-r.e. set L and r.e. sets P and Q such that:

(1) Each $\mathbf{g}_i$ is a maximal n-r.e. degree below $\mathbf{a}$ such that $\mathbf{q} \nleq \mathbf{g}_i \vee \mathbf{p}$ where $A = \bigoplus_i (G_i)$.
(2) $\mathbf{g}_i \leq \mathbf{g}_j \vee \mathbf{l}$ if and only if $i \leq_* j$.

This theorem can be strengthened at minimal cost to cover all Δ^0_2 orderings and at some greater cost to all Δ^0_3 ones.

Theorem: There are r.e. degrees $\mathbf{g}, \mathbf{p}, \mathbf{q}$, an n-r.e. degree $\mathbf{a}$ and an $n+1$-r.e. degree $\mathbf{d}$ such that:

(1) For every n-r.e. degree $\mathbf{w} \leq \mathbf{a}$, either $\mathbf{q} \leq \mathbf{w} \vee \mathbf{p}$, or $\mathbf{w} \leq \mathbf{g}$.
(2) $\mathbf{d} \leq \mathbf{a}$, $\mathbf{q} \nleq \mathbf{d} \vee \mathbf{p}$, and $\mathbf{d} \nleq \mathbf{g}$.

Alberto Marcone.
Title: The reverse mathematics of the maximal linear extension theorem for WPOs (joint work with Shore)
Abstract: A well-partial order (WPO) is a well-founded partial order with no infinite antichains. Every linear extension of a WPO is a well-order. The maximal linear extension theorem (de Jongh and Parikh, 1972) asserts that every WPO has a linear extension of maximal order type, i.e. such that all other linear extensions embed into it. This statement can be formalized in second order arithmetic. We prove that, over RCA_0, it is equivalent to ATR_0.

Alexandra Soskova.
Title: Enumeration Degree Spectra of Abstract Structures (joint work with I. N. Soskov)
Abstract: The degree spectrum $DS(\mathfrak{A})$ of a countable structure $\mathfrak{A}$ we define to be the set of all enumeration degrees generated by the presentations of $\mathfrak{A}$ on the natural numbers. The co-spectrum of $\mathfrak{A}$ is the set of all lower bounds of $DS(\mathfrak{A})$. We consider the connections between degree spectra and their co-spectra. We present variants of Selman's theorem, the minimal pair theorem and quasi-minimal degree theorem for degree spectra. A structure $\mathfrak{A}$ is called total if all presentations of $\mathfrak{A}$ are total sets. For every total structure $\mathfrak{A}$ the set $DS(\mathfrak{A})$ contains only total degrees. We prove that if $DS(\mathfrak{A})$ consists of total degrees above $\mathbf{0}'$, then there exists a total structure $\mathfrak{B}$ such that $DS(\mathfrak{B}) =$

$DS(\mathfrak{A})$. We use a generalized Jump inversion theorem for degree spectra. As an applications we receive structures with interesting degree spectra. We present a generalization of the co-spectrum based on the ω - enumeration degrees.

Alexey Stukachev.
TITLE: Aspects of Effective Model Theory and Sigma-Definability
ABSTRACT: We consider some aspects of effective model theory via the Sigma-definability in admissible sets. The talk is devoted to a group of natural open questions in this field, concerning, in particular, semilattices of Sigma-degrees and degrees of presentability of structures. For each of the questions, we discuss some partial solutions known at present and possible approaches to the general case.

Valentina Harizanov.
TITLE: C.e. and co-c.e. equivalence structures (joint work with Cenzer and Remmel)
ABSTRACT: C.e. and co-c.e. structures have been studied since the beginning of modern computable model theory. Here we focus on c.e. and co-c.e. equivalence structures. We say that an equivalence structure $\mathcal{A} = (\omega, E)$ is *c.e.* (*co-c.e.*, respectively) if its equivalence relation E is c.e. (co-c.e., respectively). We establish a number of results about the complexity of isomorphisms of c.e. and co-c.e. equivalence structures. While any c.e. equivalence structure with infinitely many infinite equivalence classes is isomorphic to a computable structure, there are c.e. equivalence structures with finitely many infinite equivalence classes, which are not isomorphic to computable structures. We show that if c.e. equivalence structures $\mathcal{A}_1$ and $\mathcal{A}_2$ are isomorphic to a computable structure $\mathcal{A}$ that is computably categorical or relatively Δ^0_2 categorical, then $\mathcal{A}_1$ and $\mathcal{A}_2$ are Δ^0_2 isomorphic. On the other hand, we construct a co-c.e. equivalence structure with all equivalence classes of sizes 1 or 2, which is not Δ^0_2 isomorphic to any c.e. equivalence structure.

Antonin Kučera.
TITLE: Demuth randomness (joint works with Nies)
ABSTRACT: Demuth tests generalize Martin-Löf tests $(G_m)_{m \in \mathbb{N}}$ in that one can exchange the m-th component for a computably bounded number of times. A set $Z \subseteq \mathbb{N}$ fails a Demuth test if Z is in infinitely many final versions of the G_m. If we only allow Demuth tests such that $G_m \supseteq G_{m+1}$ for each m, we have weak Demuth randomness. These notions are strictly between 1-randomness and 2-randomness and Demuth randomness is incomparable with weak 2-randomness.

The notion of Demuth randomness is stronger than 1-randomness yet compatible with being Δ^0_2. It is also known that all Demuth random sets are GL_1.

We show that a weakly Demuth random set can be high Δ^0_2, yet not superhigh. Both Demuth randomness and weak Demuth randomness are closed downward under Turing reducibility within the 1-random sets.

Any c.e. set Turing below a weakly Demuth random set is obviously K-trivial. We show, however, that there is a c.e. K-trivial set which is not Turing below any weakly Demuth random set.

Further, while there are weakly Demuth random sets which have Turing below c.e. K-trivial sets which are not strongly jump traceable, we show that any c.e. set Turing below a Demuth random set is strongly jump traceable.

Some comments and open questions will be presented.

David Diamondstone.
TITLE: Low LR upper bounds
ABSTRACT: We say that $A \leq_{LR} B$ if every B-random real is A-random–in other words, if B has at least as much derandomization power as A. This is what is called a "weak reducibility": it is implied by Turing reducibility, but does not imply Turing reducibility. In fact, even calling it a "reducibility" may be misleading, as LR-lower cones can be uncountable. (One example of an uncountable LR-lower cone is the cone $\{A : A \leq_{LR} 0'\}$ below $0'$.) However, the LR reducibility is a natural one for studying randomness. The K-trivials form the bottom degree in the LR degrees, just as the computable reals form the bottom degree in the Turing degrees, and K-trivials often play the role of computable sets from the point of view of randomness.

Much remains mysterious about the LR degrees. In particular, it is not even known whether they form a semilattice. (It is known that if a join exists, it cannot be the same as the join in the Turing degrees.) So there are many open structural questions. We show that given two (or even finitely many) low sets, there is a low c.e. set which lies LR above both. This is very different from the situation in the Turing degrees. Indeed, the Sacks splitting theorem gives us two low sets whose Turing degrees join to $0'$, so the fact that any finite collection of low sets has a single low c.e. upper bound in the LR degrees is quite surprising.

Wolfang Merkle.
TITLE: Basic methods for Solovay functions
ABSTRACT: Solovay functions are upper bounds for prefixfree Kolmogorov complexity that are infinitely often tight up to an additive constant. Solovay functions that are computable or can be approximated from above behave in many respects similar to prefixfree Kolmogorov complexity, which is a special case of such a function. The talk emphasizes basic methods and proof techniques for Solovay functions that have been applied in recent results obtained together with Hölzl and Krähling and with Bienvenu, Downey, and Nies.

Mariya Soskova.
TITLE: $\mathcal{K}$-pairs in the local structure of the enumeration degrees

Abstract: A pair of sets of natural numbers A and B forms a $\mathcal{K}$-pair if there exists a c.e. set W, such that $A \times B \subseteq W$ and $\overline{A} \times \overline{B} \subseteq \overline{W}$. $\mathcal{K}$-pairs are introduced by Kalimullin and used by him to prove the first order definability of the enumeration jump operator in the global structure of the enumeration degrees. He shows that the property of being a $\mathcal{K}$-pair is degree theoretic and first order definable in the global structure. In this talk we examine the properties of $\mathcal{K}$-pairs in the local structure of the enumeration degrees. We show that the class of $\mathcal{K}$-pairs of Σ^0_2 enumeration degrees is also locally definable. This is the first definability results in the local structure of the enumeration degrees. It unlocks the definability of further classes of Σ^0_2 degrees and allows us to characterize the complexity of the first order theory of the local structure. This is joint work with Hristo Ganchev.

Ekaterina Fokina.

Title: Isomorphism and Bi-Embeddability among Σ^1_1 Equivalence Relations

Abstract: We discuss possible approaches to measure and compare the complexity of equivalence relations on classes of computable structures. In particular, we will consider such natural equivalence relations as isomorphism and bi-embeddability. If a class of structures is closed under isomorphism and the set of its computable members is definable by a computable infinitary sentence, then the relations of isomorphism or bi-embeddability are Σ^1_1. We consider several reducibilities (in the context of sets and equivalence relations) that allows us to compare the complexity of such relations. In particular, we show that the isomorphism may be complete among Σ^1_1 equivalence relations on ω. For the bi-embeddability we can show more. In fact, for every Σ^1_1 equivalence relation E on ω there is a computable infinitary sentence φ such that the bi-embeddability relation on computable models of φ is equivalent (with respect to the chosen reducibility) to E. The main results are joint work with S. Friedman, V. Harizanov, J. Knight, C. McCoy and A. Montalbán. We also discuss some general facts about the structure of degrees of Σ^1_1 equivalence relations on ω under the reducibilities we have studied. This part is based on joint results with S. Friedman.

Barbara Csima.

Title: Introducing the Bounded Jump

Abstract: In recent work with Rod Downey and Selwyn Ng, we show that, for the usual Turing Jump, analogs of Sacks' and Shoenfield's jump inversion fail for both the truth table and bounded Turing (more commonly known as weak truth table) reducibilities. This gives further evidence that since the Turing jump is defined with respect to Turing reducibility, perhaps it is not always the "right" jump to consider on stronger reducibilities. Together with Bernie Anderson, we have defined what we call the *bounded jump*, a jump operator on the bounded Turing degrees. We have found that the bounded jump has many desirable properties, which I will present.