

Math 128A Midterm 1, 21 Feb 2003. J. Strain

1. Round $\pi = 3.14159\dots$ to 4 decimal digits. Evaluate the absolute and relative errors and the number of significant digits. Repeat with $\pi \times 10^{200}$.

2. Consider the iteration $x_0 = 1/2$,

$$x_{n+1} = x_n (2 - \pi x_n).$$

Evaluate $\lim_{n \rightarrow \infty} x_n$. Find the rate of convergence to the limit as $n \rightarrow \infty$.

3. Construct the quadratic polynomial $P(x)$ which interpolates

$$f(x) = \sin \frac{\pi x}{2} \quad \text{at } x = -1, 0, 1.$$

Show that

$$|f(x) - P(x)| \leq \frac{\pi^3}{72\sqrt{3}} \quad \text{for } |x| \leq 1.$$