FINAL

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Mathematical Tools for the Physical Sciences
121B
Spring 2003: Egilsson

Tuesday, May 20, 2003, from 5-8PM in 166 Barrows

Name (1%):_____.

REMEMBER TO SUPPLY ALL NECESSARY ARGUMENTS

You may use a copy of the textbook (M. Boas, Mathematical Methods in the Physical Sciences) but no other written material.

Also, only 9 problems will be graded.

You need to clearly circle two problems below that you don't want graded:

1 2 3 4 5 6 7 8 9 10 11

1 (11%) Solve the differential equation

$$y'' + 4y' + 4y = \cos(2x).$$

2 (11%) Find one nonzero solution to the differential equation

$$\sqrt{y''} = y.$$

3 (11%) Find the maximum value of the integral

$$\int_0^1 (xy + \dot{x}\dot{y})dt$$

for all paths $\gamma(t) = (x(t), y(t))$ from $\gamma(0) = (0, 0)$ to $\gamma(1) = (\sinh(1), 2\sinh(1))$.

4 (11%) Determine if the matrix

$$\begin{pmatrix} 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

is orthogonal or not.

5 (11%) Find an orthogonal matrix M and a diagonal matrix D such that

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix} = MDM^T.$$

6 (11%) Evaluate the limit

$$\lim_{n\to+\infty} \frac{\ln((2n)!)}{n\ln(2n)-n}.$$

7 (11%) Show that the elliptic function $y = dn(u)$, defined by

$$dn(\int_0^{\phi} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}) = \sqrt{1-k^2\sin^2\phi},$$

satisfies the differential equation

$$y'' = (2-k^2)y - 2y^3.$$

8. (11%) Define functions $A_n(x)$ by the equation

$$\cos(2xh - h^2) = \sum_{n=0}^{+\infty} h^n A_n(x)$$

and show that

$$A_n'' = -4A_{n-2}.$$

9 (11%) Show that

$$\sum_{n=0}^{+\infty} \frac{x^n H_n(x)}{n!} = \sum_{n=0}^{+\infty} \frac{x^{2n}}{n!}$$

where $H_n(x)$ are the Hermite polynomials.

10 (11%) Find all possible solutions of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + u$$

that can also be written as products $u = X(x)T(t)$.

11 (11%) Find the temperature distribution inside a spherical shell of inner radius 1 and outer radius 2 if the inner surface is held at 100C and the outer at 0C.