MSRI–Evans Talk

Monday, 4:10–5:00pm, 60 Evans

April 27 **Christopher Hacon**, University of Utah

Boundedness of varieties of general type

Let $X \subset \mathbb{P}^m_{\mathbb{C}}$ be a smooth algebraic variety, i.e., a complex manifold defined by polynomial equations. The main tool in the study of the geometry of X is given by the canonical bundle ω_X. Sections $s \in H^0(X, \omega_X^{\otimes r})$ may be written locally as $f \cdot (dz_1 \wedge \cdots \wedge dz_n)^{\otimes r}$ where f is holomorphic and $z_1, \ldots, z_n$ are local coordinates on X. For any $r > 0$, let $s_0, \ldots, s_{N_r}$ be a basis of $H^0(\omega_X^{\otimes r})$. The r-th pluricanonical map $\phi_r : X \dashrightarrow \mathbb{P}^{N_r}_{\mathbb{C}}$ is then given by $x \to [s_0(x) : \cdots : s_{N_r}(x)]$. We say that X is of general type if for some $r \gg 0$ the map ϕ_r is birational (an isomorphism on the complement of a closed subset).

It is well known that for cures of general type (i.e., genus ≥ 2), ϕ_r is an isomorphism for all $t \geq 3$ and that for surfaces ϕ_t defines a birational map for all $t \geq 5$.

In this talk I will discuss a similar result that holds in all dimensions:

Theorem 0.1. *For any integer $n > 0$, there exists an integer $r(n)$ such that if X is a complex projective manifold of general type and dimension n, then ϕ_t is birational for all $t \geq r(n)$.*