

MSRI–Evans Talk

Monday, 4:10–5:00pm, 60 Evans

Nov. 3 **Timothy Gowers**

Quadratic Fourier Analysis

A remarkable theorem of Szemerédi states that if δ is any positive real number and k is any positive integer, then there exists a positive integer N such that every set of at least δN integers between 1 and N must contain an arithmetic progression of length k . To put this more loosely: dense sets of integers cannot fail to contain long arithmetic progressions.

The first result in the direction of the theorem was due to Roth, who proved it for progressions of length 3 using Fourier analysis. However, this proof did not seem to be generalizable, and Szemerédi’s proof was very different, as was a subsequent very influential proof by Furstenberg. In the end, though, it turned out that it was after all possible to generalize Roth’s argument, and the generalization has led to a notion of “higher-order” Fourier analysis, of which the first new case, quadratic Fourier analysis, is now quite well developed. I shall try to explain what quadratic Fourier analysis is and what it is good for.