George M. Bergman
961 Evans Hall

Fall 1999, Math 1B

First Midterm – Make-up Exam

28 September, 1999
8:10-9:30 AM

1. (30 points, 6 points apiece) Find the following.

(a) $\int (x+1)\, e^{-x}\, dx$

(b) $\int \sin^3 x \cos^4 x \; dx$

(c) $\int_{-1}^{1} \frac{(x+1)^3}{x^2+1}\, dx$

(d) An integral expressing the area A of the surface obtained by revolving the portion of the curve $y = \sin x$ from $x = a$ to $x = b$ about the x-axis. Do not attempt to carry out the integration.

(e) $\lim_{n\to\infty} ((n^2 + 3n + 1)^{1/2} - n)$

2. (40 points, 10 points apiece) Compute the following integrals.

(a) $\int \tan^{-1}\sqrt{x}\; dx$

(b) $\int \sqrt{1+\sqrt{x}}\; dx$

(c) $\int (-6x - x^2)^{-1/2}\; dx$

(d) $\int_0^{e^{-2}} t^{-1}(\ln t)^{-4}\, dt$

3. (12 points) (a) (6 points) If a function f is continuous on $(0, 1]$, but discontinuous at 0, what is meant by $\int_0^1 f(x)\, dx$ (assuming this exists)?

(b) (6 points) Using the definition of such integrals, obtain a formula for $\int_0^1 x^{-c} dx$, where $0 < c < 1$. (Correct reasoning: 3 points; correct formula: 3 points.)

4. (18 points) (a) (9 points) State the Principle of Mathematical Induction.

(b) (9 points) Let the sequence of real numbers a_n be defined by $a_1 = 1$, $a_{n+1} = 1/(a_n + 1)$ for $n \geq 1$. Prove that for all positive integers n, $a_n = f_n/f_{n+1}$, where f_n is the nth Fibonacci number. (Recall that the Fibonacci numbers are defined by the relations $f_1 = f_2 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$.)
(Suggestion: use Mathematical Induction.)