Math 113: Introduction to abstract algebra.

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Make-up midterm, Monday, December 11, 12:30–3:30 p.m., 60 Evans.

Name:

Note. You have to do **three** out of the four problems. Cross out the problem that you don't want to be graded. *Give complete proofs of the assertions you are making and of the correctness of your answers.* Theorems proved in the book or in class may be used without proof (but do give the formulation).

1	
2	
3	
4	
Total	

Problem 1.
What is the smallest n for which there exists an element of order 12 in the alternating group A_n?

Solution:

Problem 2.
Let G be the group $\mathbb{Z} \times \mathbb{Z}_5$, and let $a = (1, 1) \in G$. What is the order of the group $G/\langle a \rangle$?

Solution:

Problem 3.

Denote by $\mathbb{C}^*$ the multiplicative group of non-zero complex numbers and define the homomorphism $f\colon \mathbb{C}^* \to \mathbb{C}^*$ by $f(z) = z/\bar{z}$, where $\bar{z}$ denotes the complex conjugate of z. (You do not have to prove that f is a homomorphism.)

Determine the image of f and the kernel of f, and prove that

$$\mathbb{C}^*/\mathbb{R}^* \cong \mathbb{T},$$

where $\mathbb{R}^*$ is the multiplicative group of non-zero real numbers and $\mathbb{T}$ is the circle group: $\mathbb{T} = \{z \in \mathbb{C} \mid |z| = 1\}$.

Solution:

Problem 4.
How many homomorphisms are there from the pcrmutation group S_3 to the Klein four group V_4?

Solution: