Ian Agol

Final, Math 1A, section 1

Wednesday, December 17, 2008, 12:30 pm - 3:30 pm

Name:

Student ID#:

DO NOT TURN THE PAGE UNTIL TOLD TO DO SO. READ THIS PAGE BEFORE YOU DO ANYTHING ELSE.

- Put personal items under your seat. *NO USE OF NOTES, TEXTS, CALCULATORS, OR EACH OTHER IS ALLOWED*
- The numbers next to each problem represent the points for the problem.
- Show your work neatly as completed. You do not need to rewrite the question, but you should make it clear which problem you are working on, writing each problem on a separate page. You may use scratch paper to work out your problem, but you will only get credit for the answer on the page. Partial credit will be given for quality work towards solutions. On the other hand, unsupported answers will not receive full credit.
- If you finish early, raise your hand to turn in the exam. Otherwise, remain seated until the end of the exam. If you use a crib sheet, turn it in with your exam, placing the exam and the crib sheet in the front of your blue book.
- **STOP** means **STOP**. When you are told that time is up, put your pencils down, and pass your completed exam to the front per the instructions of the proctors. **Remain seated until all the tests are passed in.**
- Read the following and sign your name.

  I affirm that I will give nor receive unauthorized assistance on this examination. All the work that appears on the following pages is solely my own.

Signature: ______________________

problem	points	score
1.	15	
2.	20	
3.	25	
4.	10	
5.	15	
6.	30	
7.	45	
8.	20	
9.	20	
total	200	

15 1. A ladder $10ft.$ long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of $2ft./s.$, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is $6ft.$ from the base of the wall?

20 2. Prove that $x\ln(x) \geq x - 1$ for $x > 0$.

25 3. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining two edges. Find the maximal capacity of such a cup in terms of R. Justify your answer, but you may use the formula for the volume of a cone, you do not need to derive it.

10 4. Find $\frac{d}{dx}\int_{x^2}^{1+x^2} \ln(t)\, dt$ for $x > 0$, justifying your answer.

15 5. Show that the tangent lines to the curves $x = y^3$ and $y^2 + 3x^2 = 5$ are perpendicular when the curves intersect. Justify your answer.

30 6. Evaluate the following integrals, justifying your answers:

   (a) $\int_0^1 x - \frac{\tan^{-1} x}{1+x^2} dx$

   (b) $\int_0^2 \sqrt{4 - x^2} dx$

   (c) $\int e^x \sqrt{1 + e^x} dx$

45 7. For the function $f(x) = e^x/x$, find with justification

   (a) the domain

   (b) intercepts

   (c) symmetry

   (d) asymptotes

   (e) intervals of increase or decrease

   (f) local maximum and minimum values

   (g) concavity and points of inflection

   (h) Then sketch the graph $y = f(x)$, marking on your graph all of the information you have found.

20 8. Let $f(x) = x - 2\sqrt{x}$.

   (a) Prove that f is increasing for $x > 1$.

   (b) Find an inverse function for $f(x)$ on the interval $x > 1$.

   (c) Prove rigorously the following limit, using the precise definition of an infinite limit:

$$\lim_{x \to \infty} x - 2\sqrt{x} = \infty$$

20 9. Suppose you make napkin rings by drilling holes through the centers of balls with different diameters and different sized holes. Suppose that the napkin rings have the same height h. Show that the volumes of the napkin rings are the same. Justify your answer.