Math 104: Midterm 1
Prof. Beth Samuels
October 4, 2005

Name :

Student ID Number:

Instructions: This is a closed-book test. Each problem is worth 20 points. Read the questions carefully, and show all your work. All work should be done on the exam paper. Additional white paper is available if needed. Good luck.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	

Let X be a metric space, and let E be a set in X.

(1) Let $\mathbb{F}_2$ be the field of two elements $\{0, 1\}$ with the property that $1 + 1 = 0$. Show that $\mathbb{F}_2$ cannot be turned into an ordered field.

(2) (a) Define **an upper bound** of E and **the least upper bound** of E.
    (b) Give an example to show that $\mathbb{Q}$ does not have the least upper bound property.

(3) (a) Define **a limit point** of a set E and a **closed** set E.
     (b) Is $\mathbb{N}$ a closed subset of $\mathbb{R}$? Prove your answer.

(4) Let $\{E_n\}$ be a finite or infinite collection of sets E_n. Prove $(\cup E_n)^c = \cap(E_n^c)$.

(5) (a) Define a **convergent sequence** $\{s_n\}$.

   (b) Compute and explain the following limits: (If the sequence converges, find N explicitly in terms of ϵ.)

   i): $\lim\limits_{n\to\infty} \dfrac{1}{n}$

   ii): $\lim\limits_{n\to\infty} (n + 1/n)$

   iii): $\lim\limits_{n\to\infty} (\dfrac{1}{n})(n)$

(6) Prove that if the sequence $\{s_n\}$ converges, then $\{s_n\}$ is bounded.