MATHEMATICS 104, FIRST MIDTERM

FEBRUARY 14, 2003. PROFESSOR H. WU

1. Assume the triangle inequality for real numbers.
   (a) (10%) Prove: $|a| - |b| \leq |a - b|$ for all $a, b \in \mathbb{R}$.
   (b) (15%) Prove by induction that for all $n \geq 1$ and for all a_0, a_1, $\ldots$, a_n in $\mathbb{R}$,

$$|a_0| - |a_1| - \cdots - |a_n| \leq |a_0 - a_1 - \cdots - a_n|$$

2. (15%) Prove that $(97 - \sqrt{2})^{1/16}$ is not a rational number.

3. (30%) Let A be a nonempty bounded subset of $\mathbb{R}$ and let B be the set of all numbers of the form $a - 15$, where $a \in A$. Prove directly, without invoking any homework problems, that $\sup B = \sup A - 15$.

4. (30%) Determine the limit of the sequence

$$a_n = \frac{1 + \cos n}{2 + \sqrt{n}}$$

and prove your claim.