

QUALIFYING EXAMINATION SYLLABUS

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Date: August 19th 2005.

1. MAJOR TOPIC: ALGEBRAIC NUMBER THEORY

- Number fields. Norms and traces. Integrality.
- Local fields. DVRs. Valuations/absolute values. Hensel's lemma. Completions.
- Discriminants and differentials.
- Decomposition and inertia groups. Ramification.
- Dirichlet's unit theorem and finiteness of the class group.
- Product formula.
- Quadratic fields. Cyclotomic fields. Quadratic reciprocity.
- Statement of the main theorems (without proof) of
 - Local class field theory
 - Global class field theory (ideal/ idèlic versions)

References: Neukirch, *Algebraic Number Theory*.

2. MAJOR TOPIC: ANALYTIC NUMBER THEORY

- Tate's thesis:
 - Background: Pontryagin duality. Fourier analysis on locally compact topological groups (without proof).
 - Restricted direct products. Adèles. Idèles. Idèle class groups. Generalized ideal class groups. Hecke characters.
 - Local ζ -functions and functional equations.
 - Poisson's summation formula/ Riemann-Roch.
 - Global ζ -functions, Analytic continuation, functional equations.
 - Hecke L -functions.
 - Analytic class number formula
- Riemann zeta function. The prime number theorem (sketch).
- Primes in arithmetic progression. Chebatorev density theorem.

References: Ramakrishnan & Valenza, *Fourier Analysis on Number Fields*.
Tate, *Fourier Analysis in Number Fields and Hecke's Zeta Functions*.

3. MINOR TOPIC: COMPLEX ANALYSIS

- Holomorphic functions. Cauchy Riemann equations. Laurent series.
- Cauchy's theorem. Cauchy's formula. Morera's theorem. Fundamental theorem of algebra.
- Singularities. Taylor's theorem. Maximum principle. Liouville's theorem. Schwarz's lemma.
- Residue theorem. Argument principle. Rouché's theorem.
- Conformality. Linear fractional transformations. Riemann mapping theorem. Open mapping theorem.

Reference: Ahlfors, *Complex Analysis*.