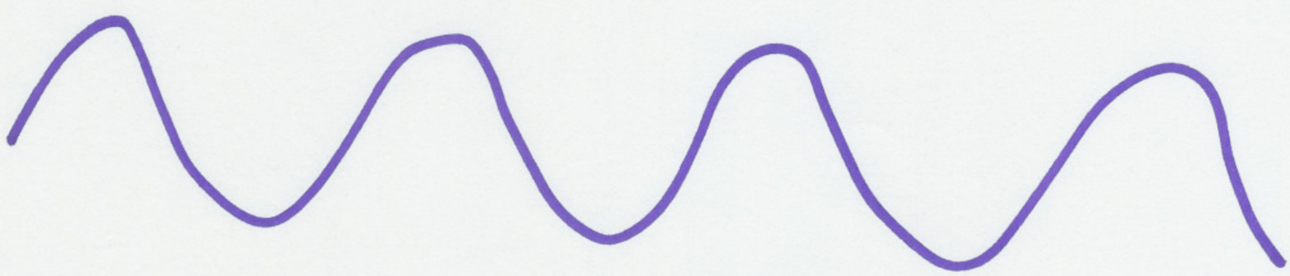
SEMINAR 4

FOURIER TRANSFORM

~ 1800

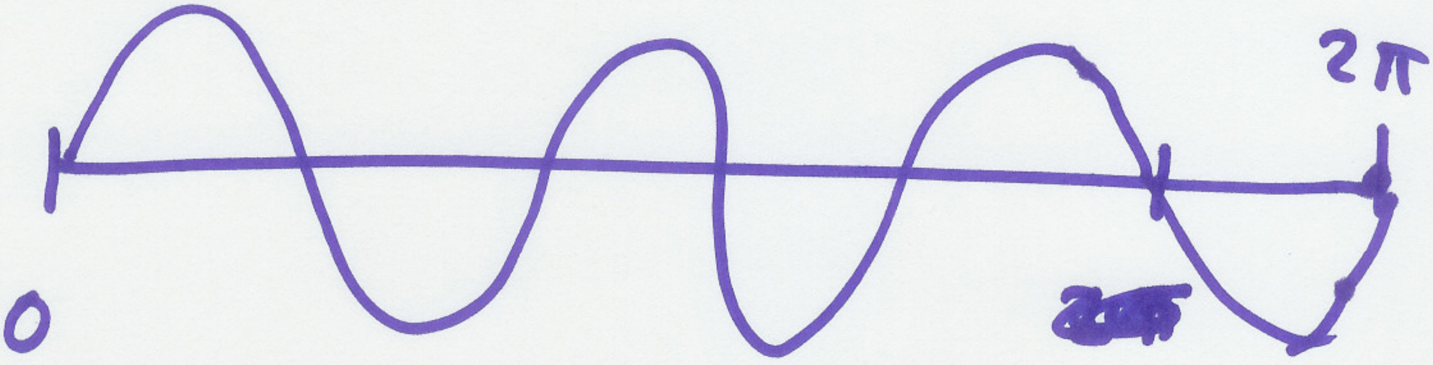
(while Fourier was in Egypt with Napoleon)

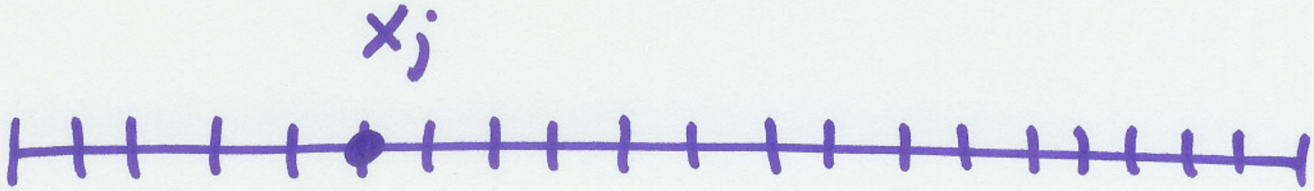
wave/signal ⇝ its frequencies



$$\cos kx = \mathrm{Re}\,(e^{ikx})$$

$$= \frac{1}{2}\left(e^{ikx} + e^{-ikx}\right)$$





DISCRETIZE

$\rightsquigarrow$ N numbers

$$x_j = \frac{2\pi j}{N}, \quad j = 0, 1, \dots, N-1$$

$$f_j = e^{\frac{2i\pi k j}{N}}$$ value of e^{ikx} at x_j

③

$$(f_0, \dots, f_{N-1}) \to (\hat{f}_0, \dots, \hat{f}_{N-1})$$

FOURIER TRANSFORM

- WHERE

$$\hat{f}_j = \begin{cases} 0 & j \neq 5 \\ 10 & j = k \end{cases}$$

FOURIER TRANSFORM TELL US AT WHAT FREQUENCIES WAVE/SIGNAL

LIVES ...

④ (See MATLAB Examples ON-LINE)

LENGTH

$$\| f \| = \left(\sum |f_j|^2 \right)^{\frac{1}{2}}$$

$$\| \hat{f} \| = \left(\sum |\hat{f}_j|^2 \right)^{\frac{1}{2}}$$

<u>THEOREM</u> $\| f \| = \| \hat{f} \|$

F.T. AS A MATRIX

$$\omega = e^{-\frac{2\pi i}{N}}$$

$$F = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & \cdots & \cdots & 1 \\ 1 & \omega & \omega^2 & \vdots \\ \vdots & \omega^2 & \omega^{2\cdot 2} & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{(N-1)(N-1)} \end{pmatrix}$$

DON'T WORRY ABOUT IT....