SEMINAR 11

Fun website:

sepwww.stanford.edu/oldsep/hale/FftLab.html

OR

Google "FFT Lab"

$$f = \begin{pmatrix} f_1 \\ \vdots \\ f_N \end{pmatrix} \xrightarrow{F} \hat{f} = \begin{pmatrix} \hat{f}_1 \\ \vdots \\ \hat{f}_N \end{pmatrix}$$

$$\hat{f}_k = \frac{1}{\sqrt{N}} \sum \omega_N^{-jk} f_j$$

$$\omega_N = e^{\frac{2\pi i}{N}}$$

How expensive is Fourier tranform?

$$f_j \rightsquigarrow \omega_N^{-jk} f_j$$

N^2 complex multiplicatlins

$f_j \rightsquigarrow f_k$ $\quad N \times (N-1)$

complex additions

TOTAL COST

$$N(2N-1) \approx 2N^2$$

Lower the cost for

$$N = 2^k$$

using

$$\left(e^{\frac{2\pi i}{N}}\right)^2 = e^{\frac{2\pi i}{N/2}}$$

$$\boxed{\omega_N^2 = \omega_{N/2}}$$

DROP $1/\sqrt{N}$ (easier to write)

$N = 2 \qquad \omega_2 = -1 \qquad \omega_2^{-1} = \overline{\omega_2}$

$\hat{f}_1 = \overline{\omega_2}^{\,1} f_1 + \overline{\omega_2}^{\,2} f_2$

$\hat{f}_2 = \overline{\omega_2}^{\,2} f_1 + \overline{\omega_2}^{\,4} f_2$

$\boxed{\begin{array}{c} 4+2\times 1 \\ = 6 \\ \text{operations} \end{array}}$

$N = 4 \qquad \omega_4^2 = \omega_2 \ , \ \omega_4 = e^{\frac{\pi i}{2}} = i$

$\hat{f}_1 = \overline{\omega_4} f_1 + \overline{\omega_4}^{\,2} f_2 + \overline{\omega_4}^{\,3} f_3 + \overline{\omega_4}^{\,4} f_4$

$= \overline{\omega_4}^{\,-1} \left(\overline{\omega}_2 f_1 + \overline{\omega}_2^{\,2} f_3 \right)$

$+ \left(\overline{\omega}_2 f_2 + \overline{\omega}_2^{\,2} f_4 \right)$

$$f^o = \begin{pmatrix} f_1 \\ f_3 \end{pmatrix}, \quad f^e = \begin{pmatrix} f_2 \\ f_4 \end{pmatrix}$$

$$\hat{f}_1 = \omega_4 \hat{f}^o_1 + \hat{f}^e_1$$

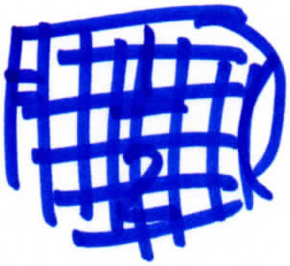
$$\hat{f}_2 = \omega_4^2 \hat{f}^o_2 + \hat{f}^e_2$$

$$\hat{f}_3 = \omega_4^3 \hat{f}^o_1 + \hat{f}^e_2$$

$$\hat{f}_4 = \omega_4^4 \hat{f}^o_1 + \hat{f}^e_2$$

# of operations

$$\begin{array}{ll} f^o \to \hat{f}^o & \\ f^e \to \hat{f}^e & \\ 4 \text{ mult} & \\ + 4 \text{ addition} & \end{array} \left.\begin{array}{c} 6 \\ 6 \\ 4 \\ 4 \end{array}\right\} 20$$



Compare to

$$4 \times 4 + 3 \cdot 4 = \underline{\underline{\underline{28}}}$$

by simple method ....

$$N \rightsquigarrow 2N$$

$$\hat{f}_k = \sum_{j=1}^{2N} \bar{\omega}_{2N}^{jk} f_j$$

$$= \sum_{\ell=1}^{N} \bar{\omega}_{2N}^{k(2\ell)} f_{2\ell}$$

$$+ \sum_{\ell=1}^{N} \bar{\omega}_{2N}^{k(2\ell-1)} f_{(2\ell-1)}$$

$$\hat{f}_k = \sum_{l=1}^{N} \bar{\omega}_N^{kl} f_{2l}$$

$$+ \sum_{l=1}^{N} \bar{\omega}_{2N}^{-k} \bar{\omega}_N^{kl} f_{2l-1}$$

$$= \hat{f}^e_k + \omega_{2N}^{k} \hat{f}^o_k$$

$$f^e = \begin{pmatrix} f_2 \\ \vdots \\ f_{2N} \end{pmatrix}, \quad f^o = \begin{pmatrix} f_1 \\ f_3 \\ \vdots \\ f_{2N-1} \end{pmatrix}$$

DIVIDE &

CONQUER !