Journeés EDP, Evian-les-Bains

Soliton Scattering by Delta Impurities

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Not surprisingly, the behaviour depends on the relation between v and q. We take $q > 0$ in this talk (more on $q < 0$ later).

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It is not clear if these limits exist!

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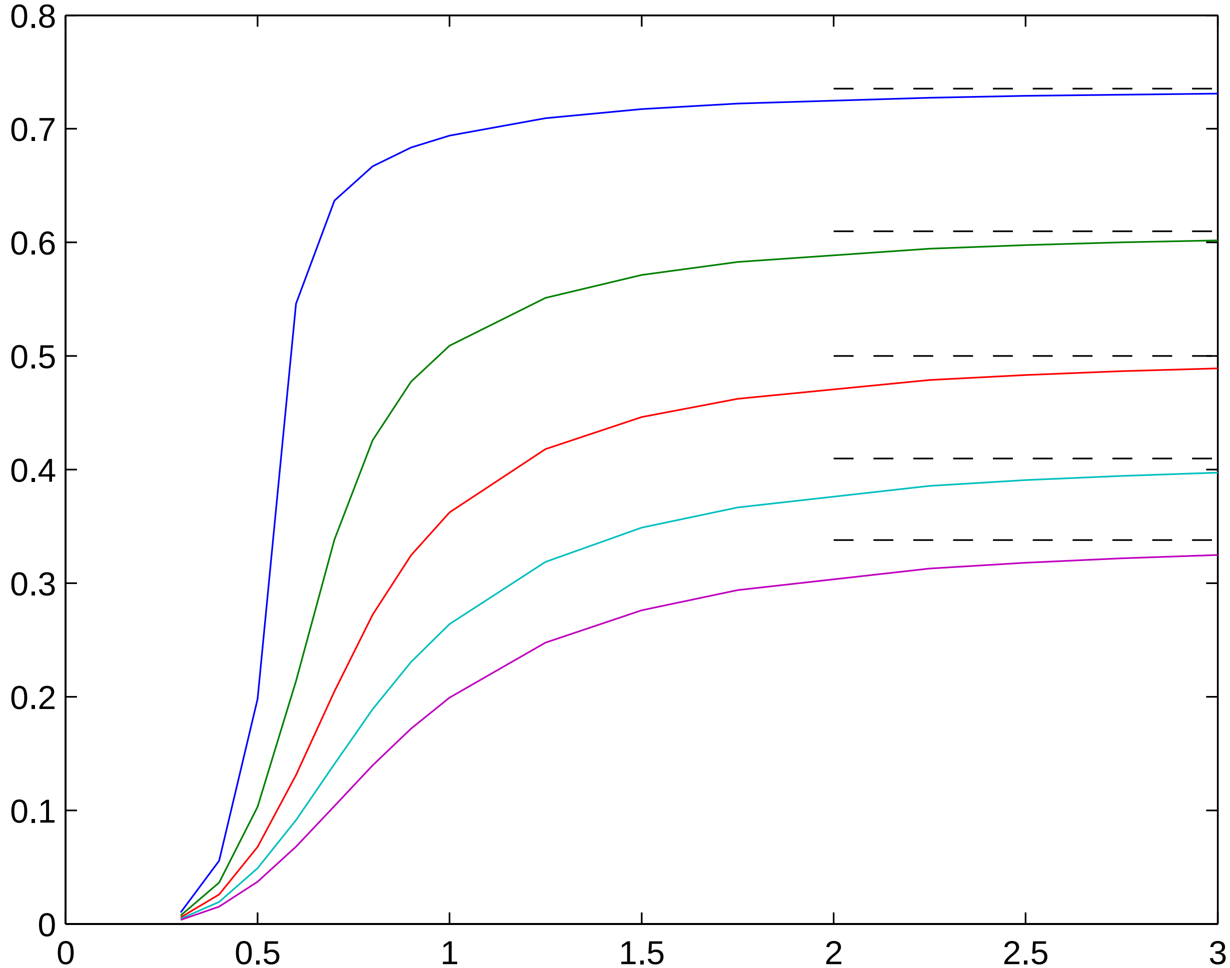
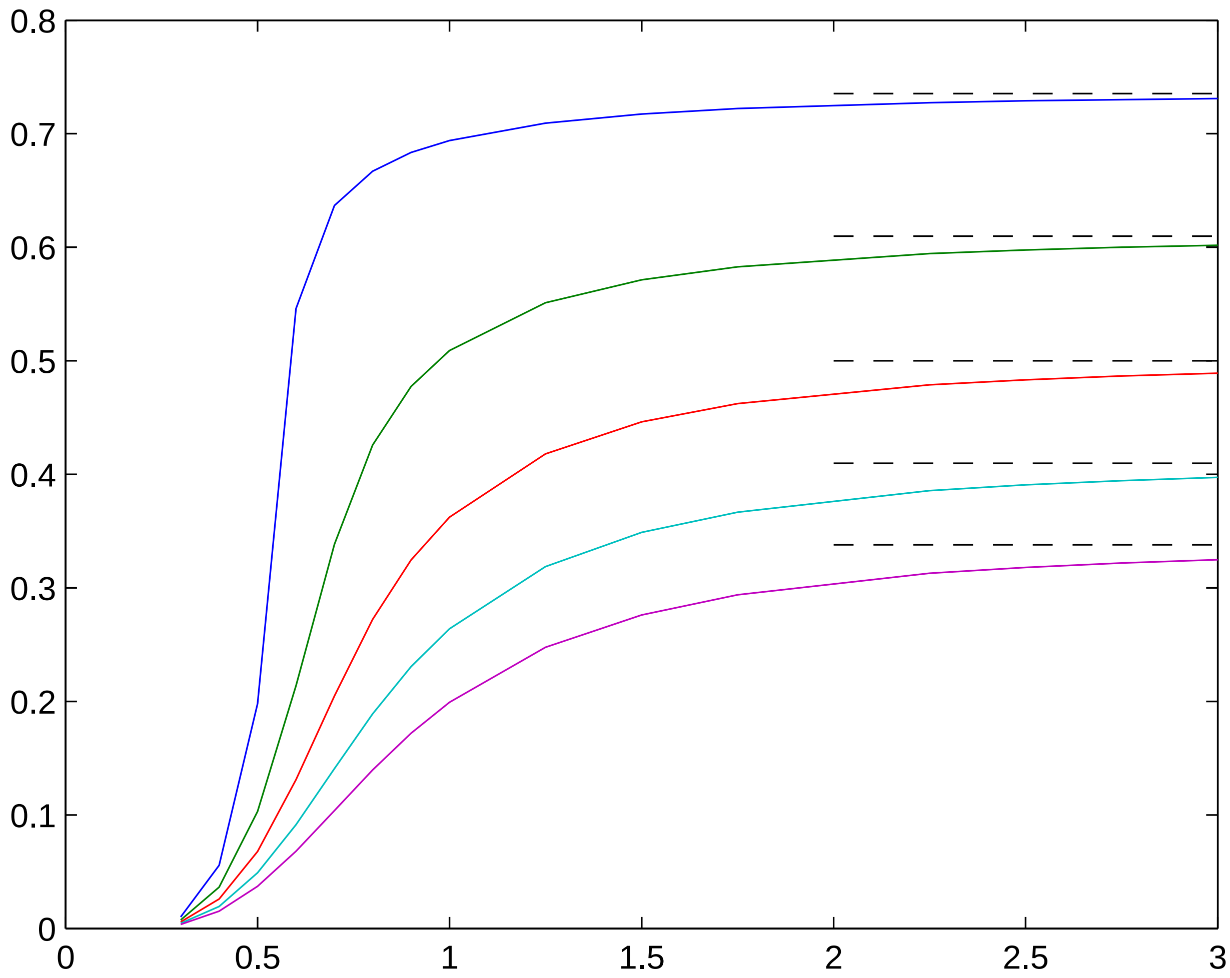
More precisely, for $v > v_0$, $v_0 = v_0(q/v,\delta)$, we have the uniform bound above.



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3



- blue: $q/v = 0.6$, and theoretical asymptotic 0.7353
- green: $q/v = 0.8$, and theoretical asymptotic 0.6098
- red: $q/v = 1.0$, and theoretical asymptotic 0.5000
- light blue: $q/v = 1.2$, and theoretical asymptotic 0.4098
- purple: $q/v = 1.4$, and theoretical asymptotic 0.3378

v	N_1	N_C	N_t	$T_q^{\mathrm{sol}}(v)$	$\lvert T_q^{\mathrm{sol}}(v) - \lVert u(t)\rvert_{x>0}\rVert_2^2\rvert$
0.50	2000	500	2000	0.067885	0.000006272
1.0	2000	500	2000	0.362334	0.000019881
1.50	2000	500	2000	0.446162	0.000027733
2.0	2000	500	2000	0.472210	0.000009639
2.50	2000	500	2000	0.483348	0.000002937
3.00	2000	500	2000	0.489011	0.000001065

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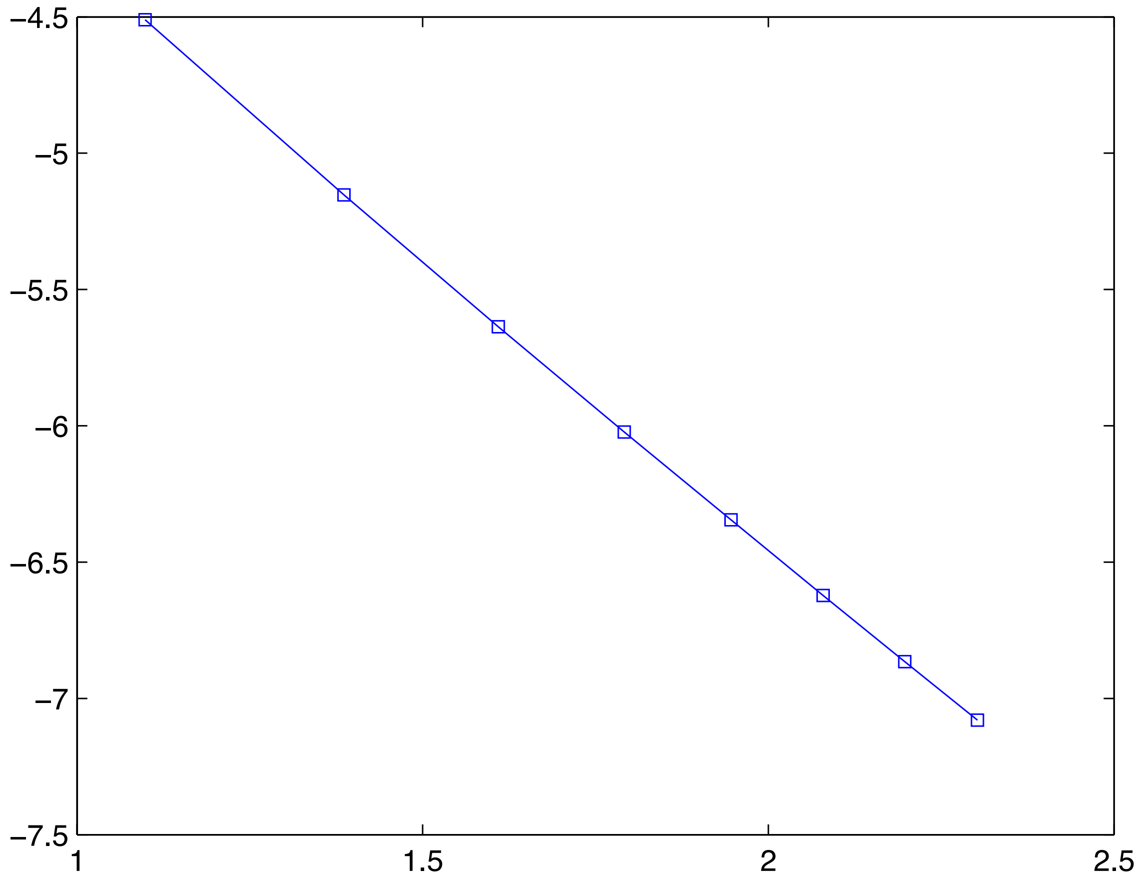
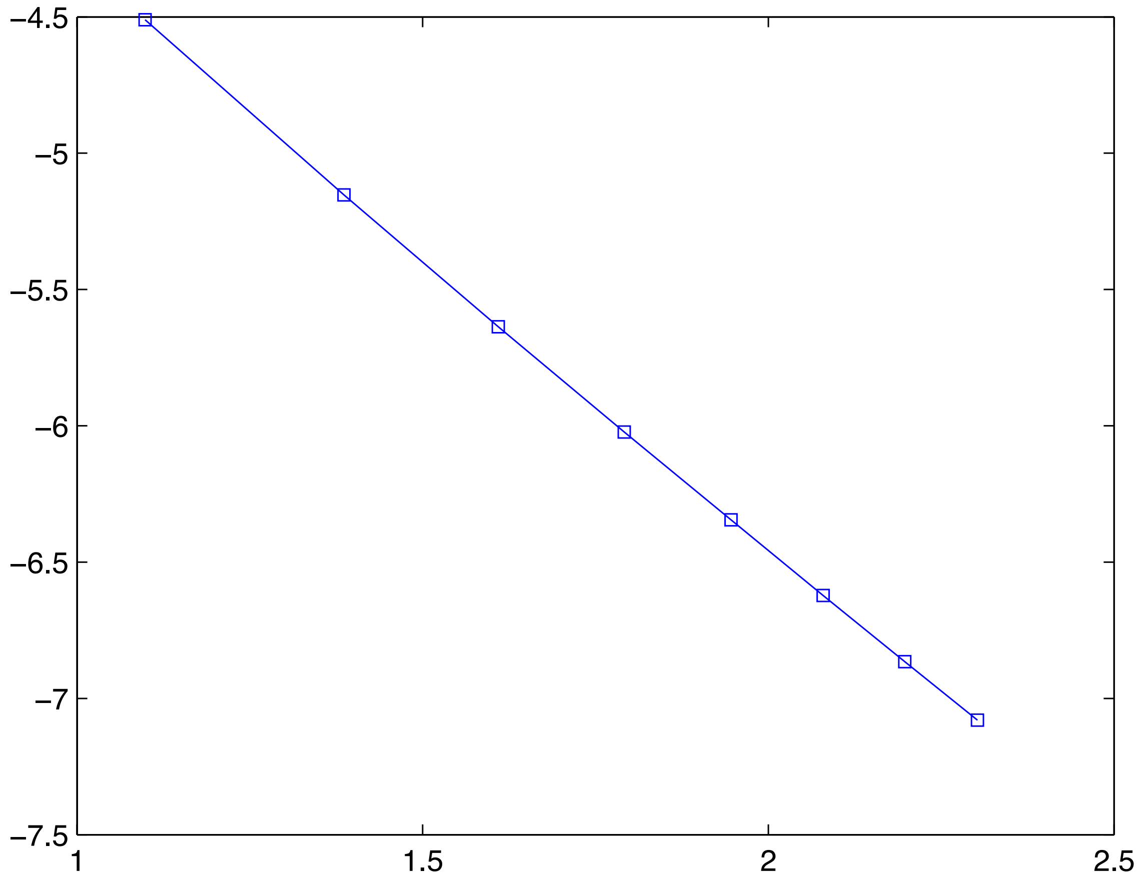
$N_\bullet$ are the number of grid points, in space, near the delta singularity, and in time, respectively.


−4.5
−5
−5.5
−6
−6.5
−7
−7.5
1
1.5
2
2.5



A plot of $\log(\frac{1}{2} - T_q^{\mathrm{s}})$ versus $\log v$ in the case $q = v$, for data at velocities $v = 3, 4, 5, \ldots, 9, 10$. The slope of this line is -2, showing that the asymptotic agreement is $(\frac{1}{2} - T_q^{\mathrm{s}}) \sim v^{-2}$.

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$$A_T(v/q) = (2|t_q(v)| - 1)_+\,,$$

and similarly for the reflected term u_R.

When $2|t_q(v)| = 1$ or $2|r_q(v)| = 1$, the L^∞ error becomes,

$$\mathcal{O}_{L^\infty_x}(\log t/\sqrt{t})\,.$$

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$$\varphi(\alpha) = \int_0^\infty \log\left(1 + \frac{\sin^2 \pi\alpha}{\cosh^2 \pi\zeta}\right) \frac{\zeta}{\zeta^2 + (2\alpha - 1)^2} d\zeta\,,$$

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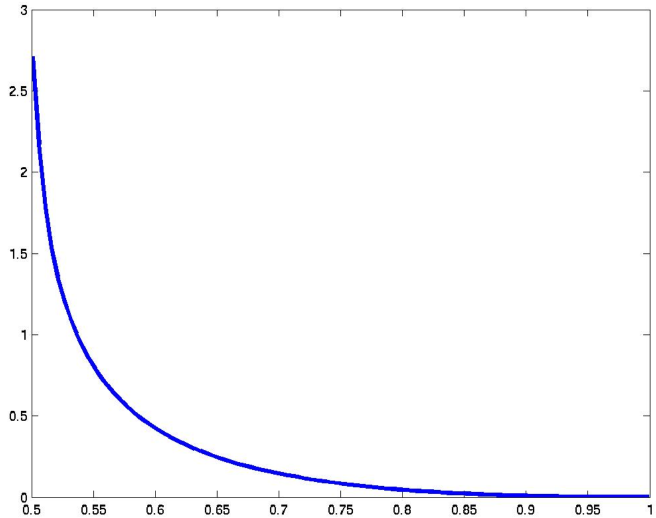

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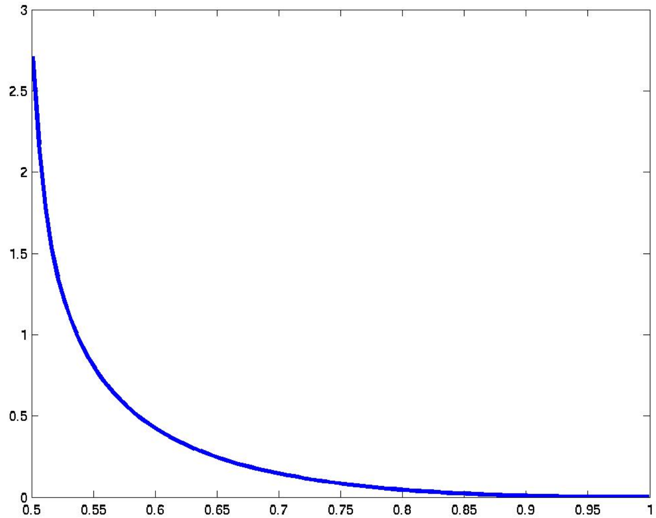

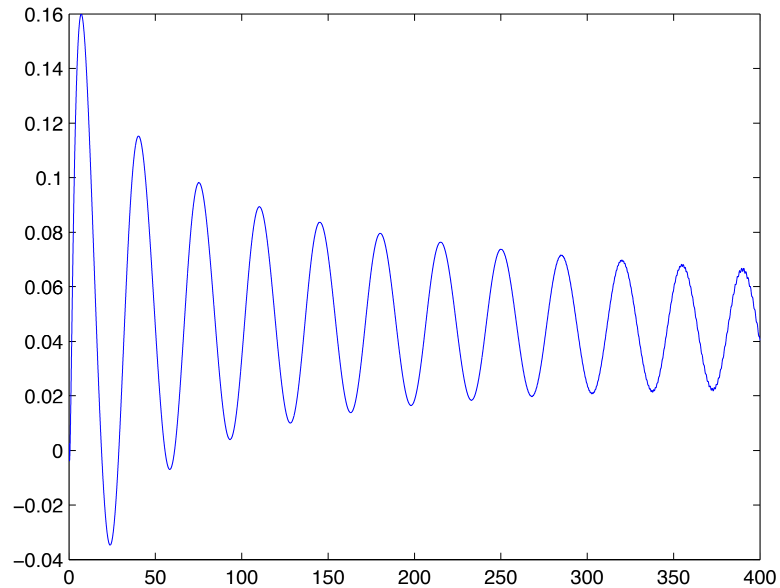

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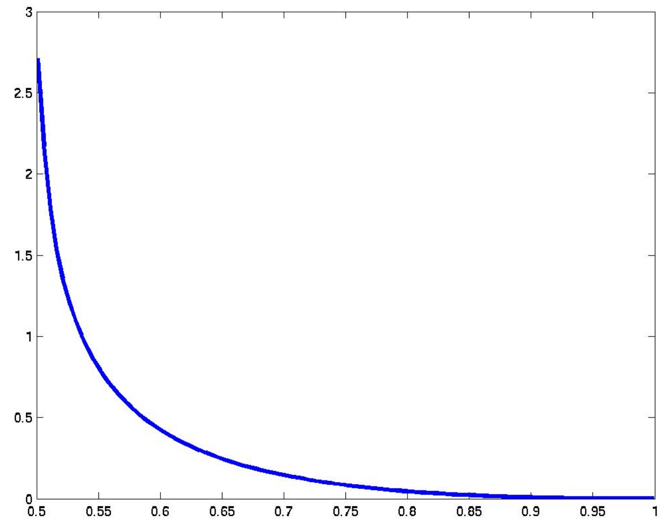
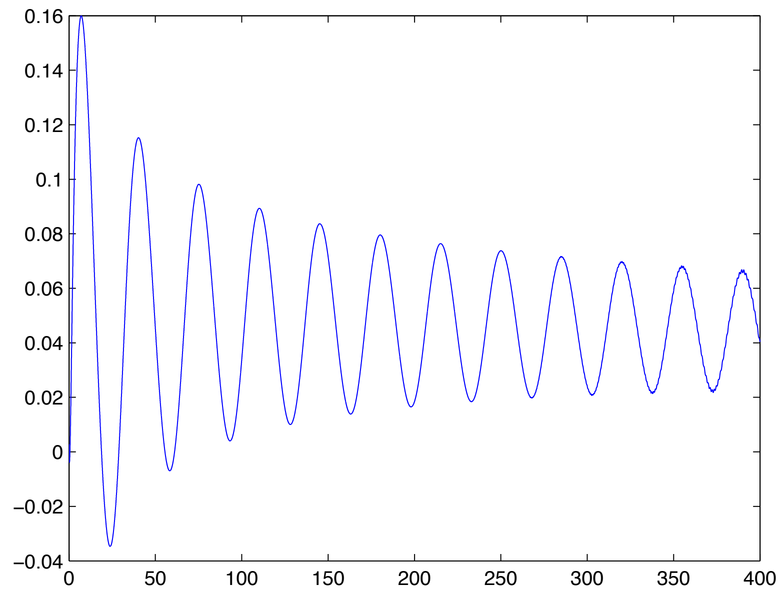
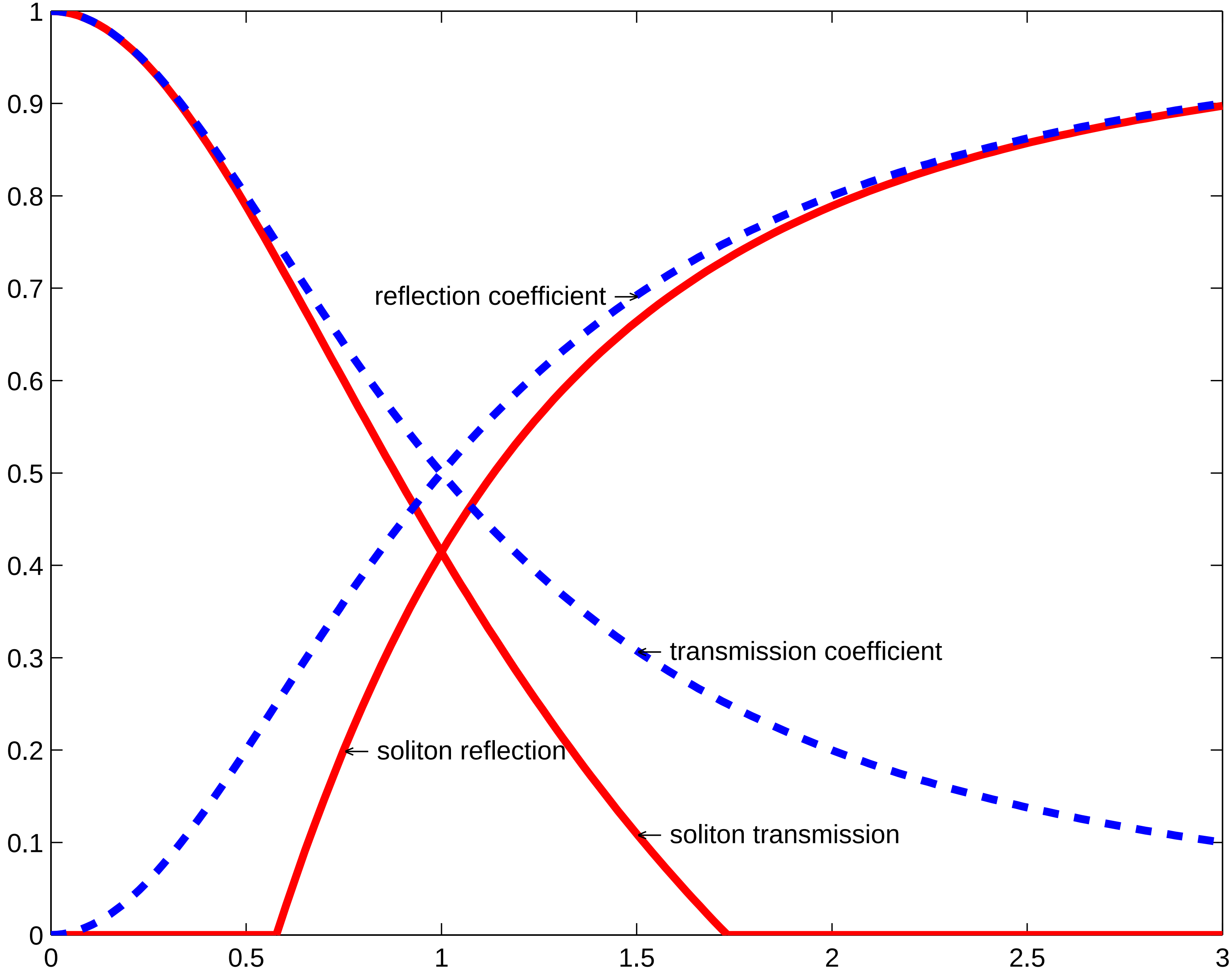




Notice that the plot on the right appears to be slowly converging to $\varphi(0.8) \simeq 0.045$. This plot represents the difference of two numbers of size ~ 100 by the end of the computation, and must therefore be taken with a grain of salt. A nice bottle of wine for a nice expression for this integral!



1
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0.6
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0.4
0.3
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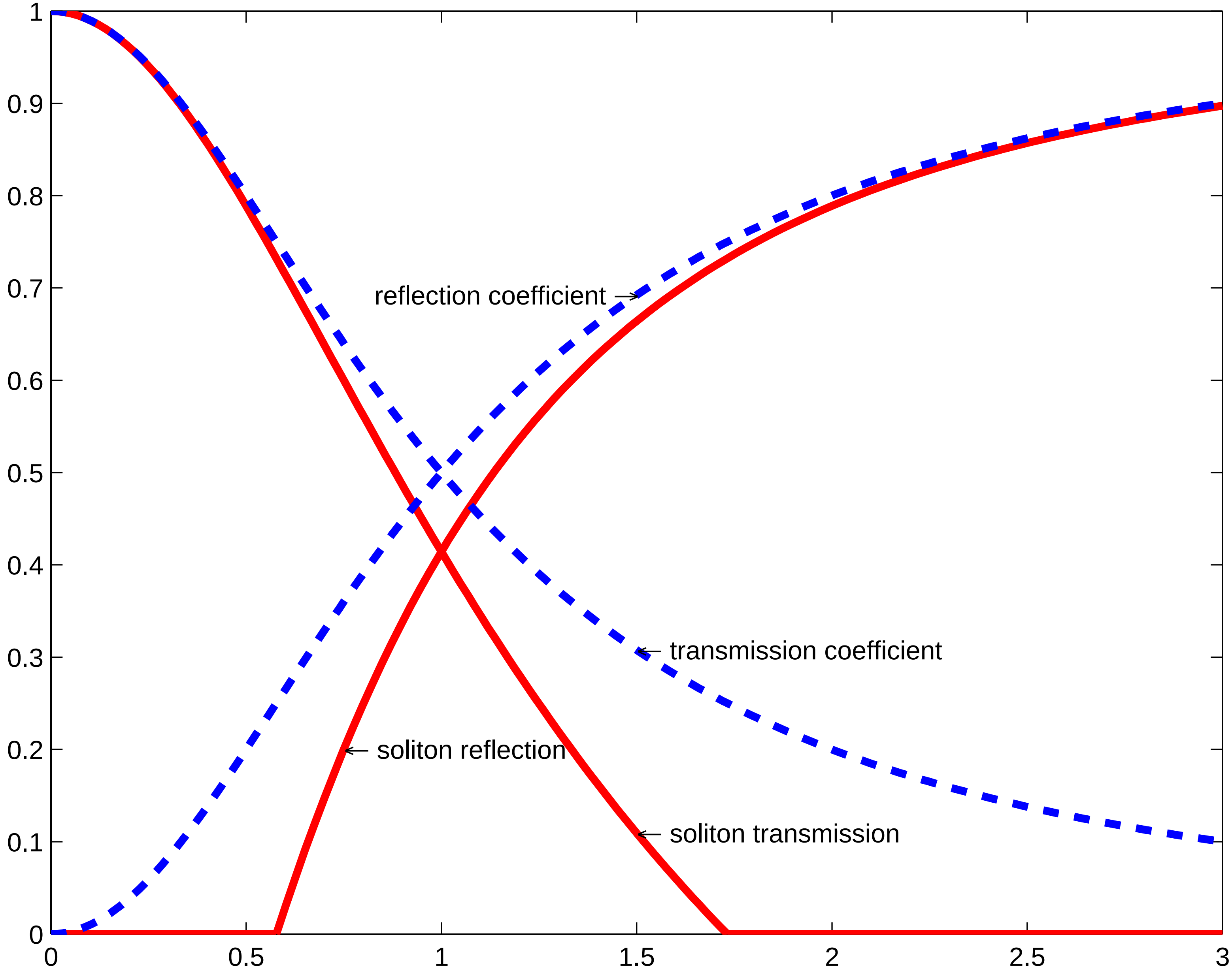
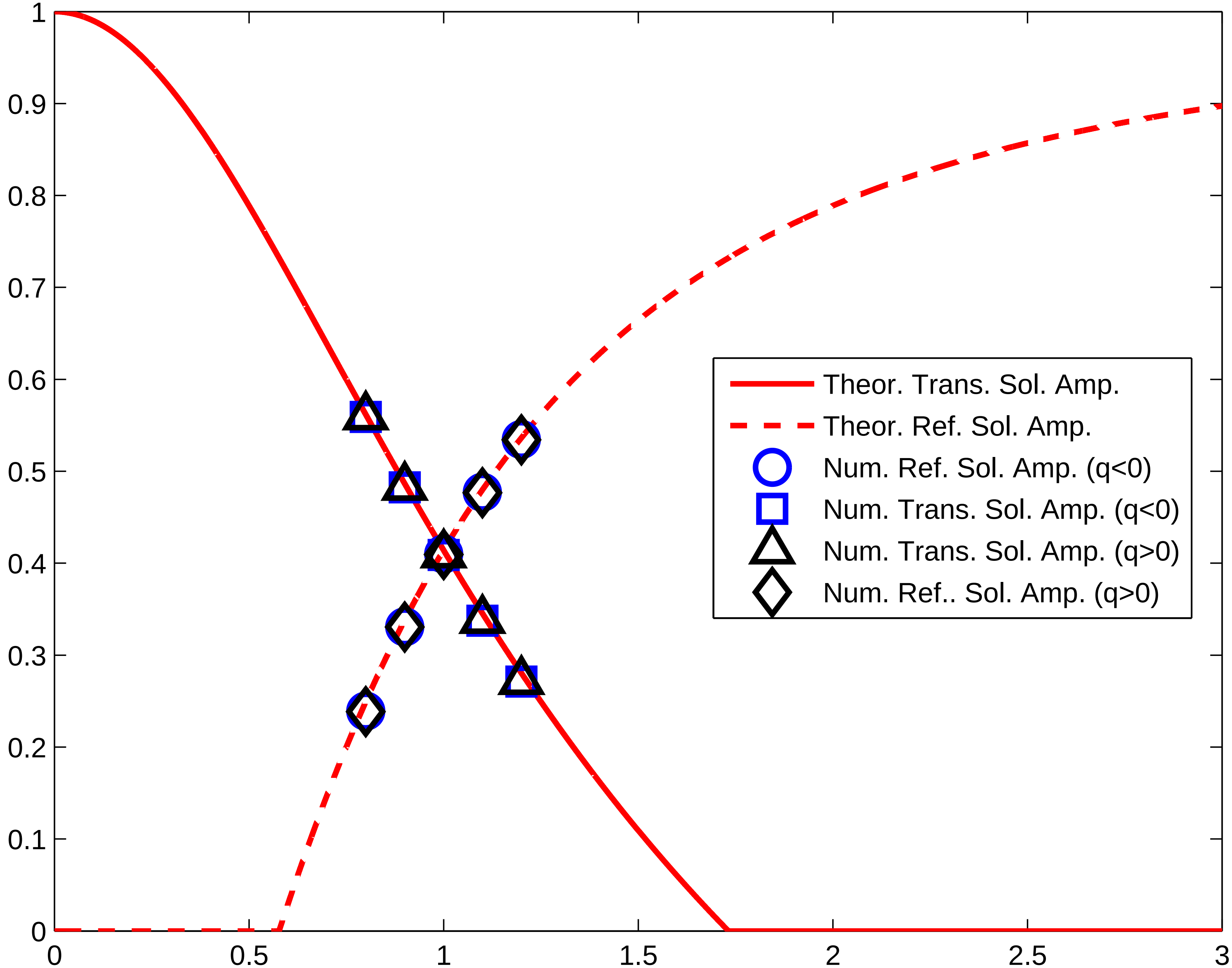
2
2.5
3
reflection coefficient
transmission coefficient
soliton reflection
soliton transmission



Soliton scattering rates compared with quantum scattering rates.



1
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2
2.5
3
Theor. Trans. Sol. Amp.
Theor. Ref. Sol. Amp.
Num. Ref. Sol. Amp. (q<0)
Num. Trans. Sol. Amp. (q<0)
Num. Trans. Sol. Amp. (q>0)
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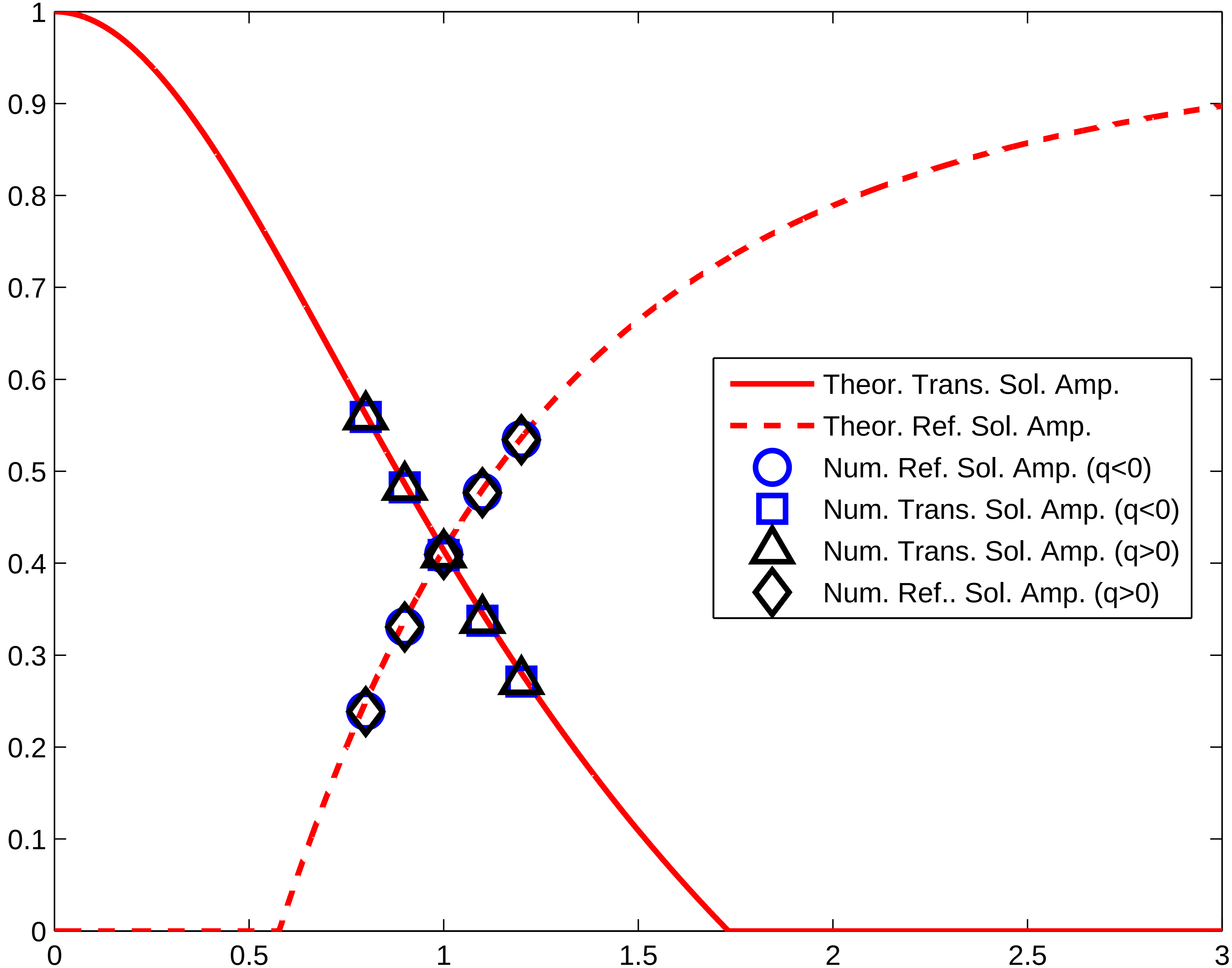

Theor. Trans. Sol. Amp.
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0
0.1
0.2
0.3
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0.5
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1.5
2
2.5
3

The proof starts with the following crucial observation:

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Since "$\delta_0 \in L_x^1$" we can take $f = g(t)\delta_0(x)$ and use $\|g\|_{L_t^{4/3}}$ on the right hand side.

Phase 1 (Pre-interaction).

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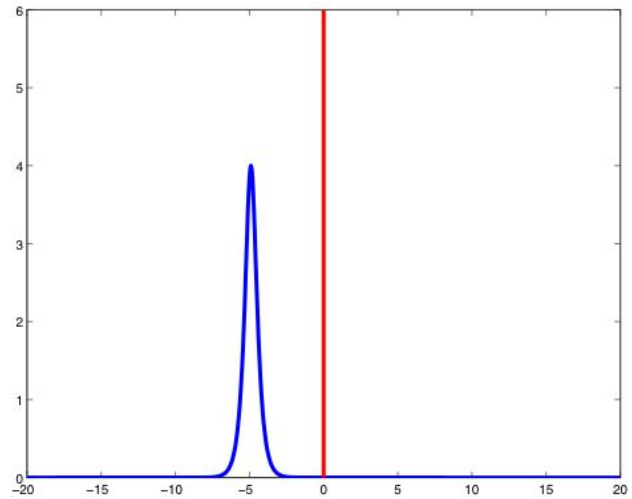
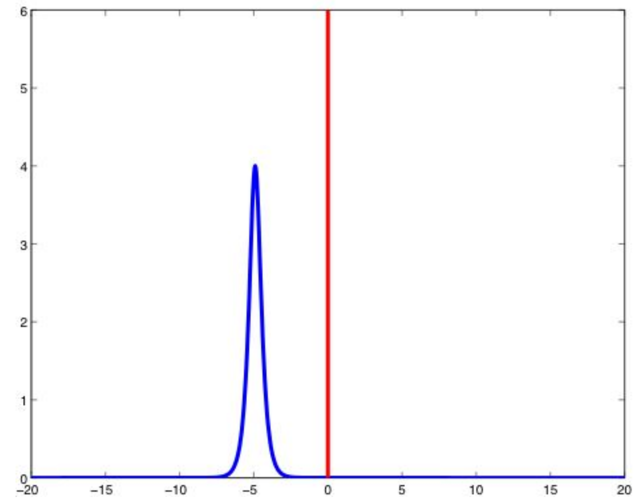
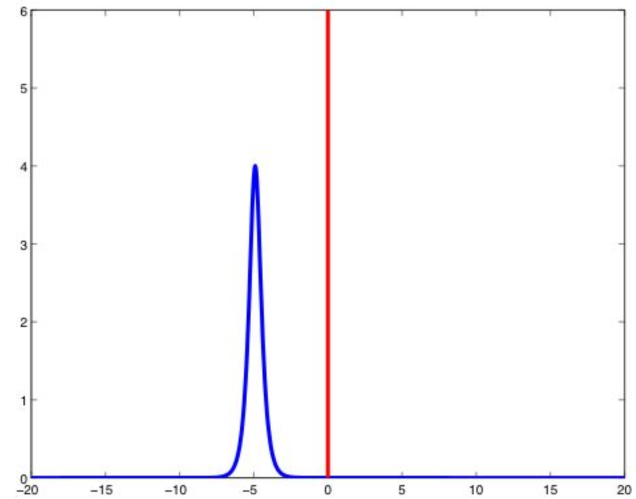




Phase 1 (Pre-interaction).

$$0 \le t \le t_1,\ t_1 = |x_0|/v - v^{-\delta}$$



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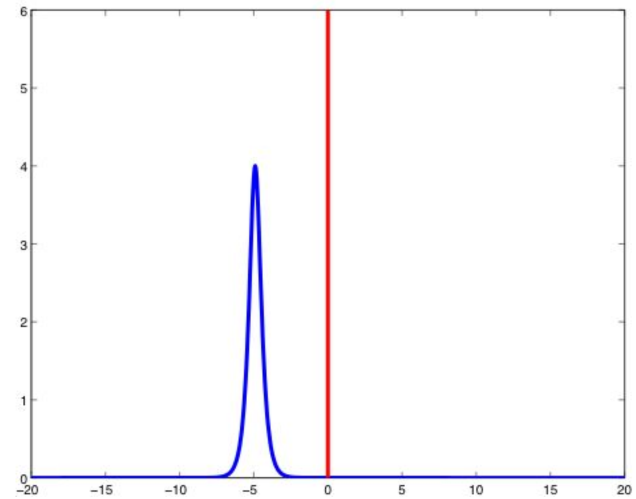

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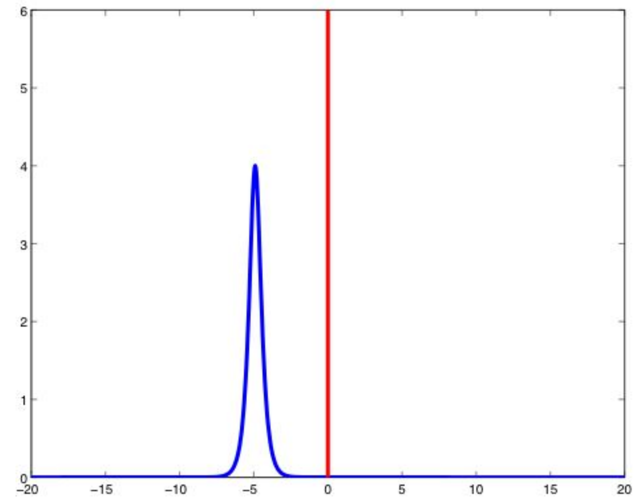

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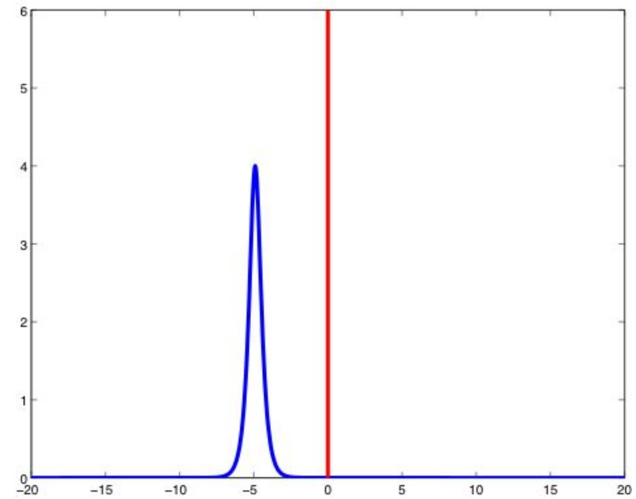

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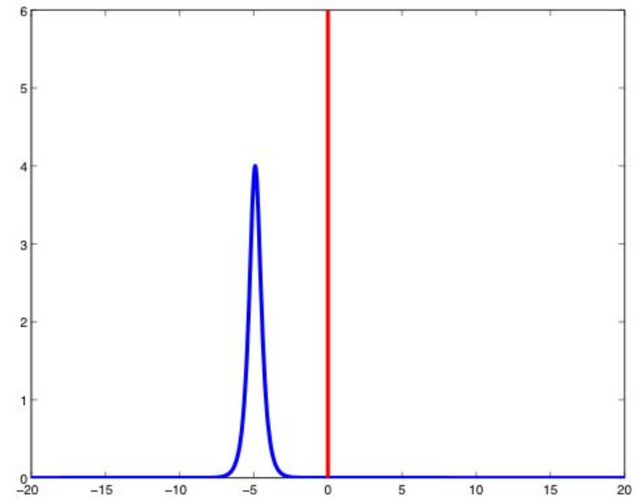

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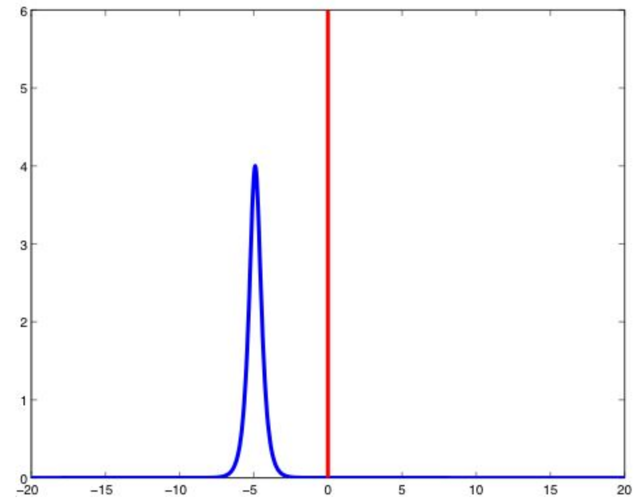

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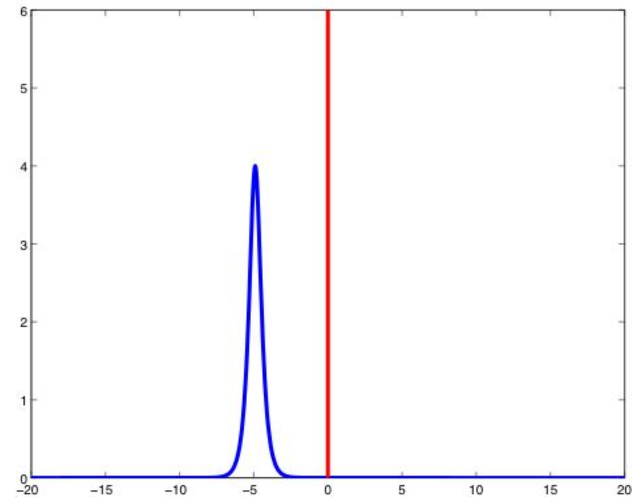

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$$qe^{-v^{1-\delta}} \leq v^{-1/2} \leq v^{-\delta/2}\,.$$

Phase 2 (Interaction).

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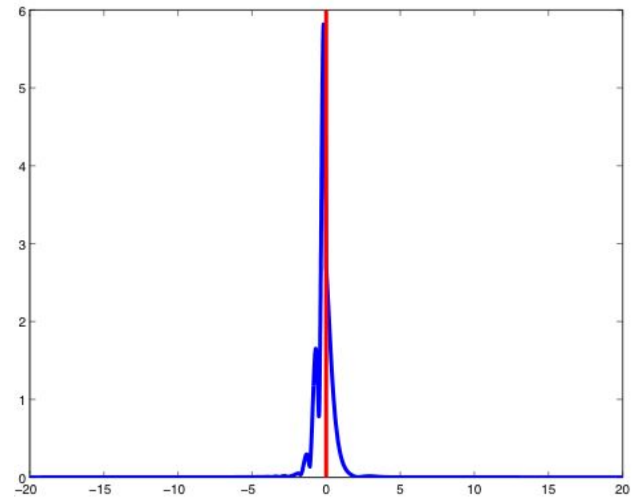

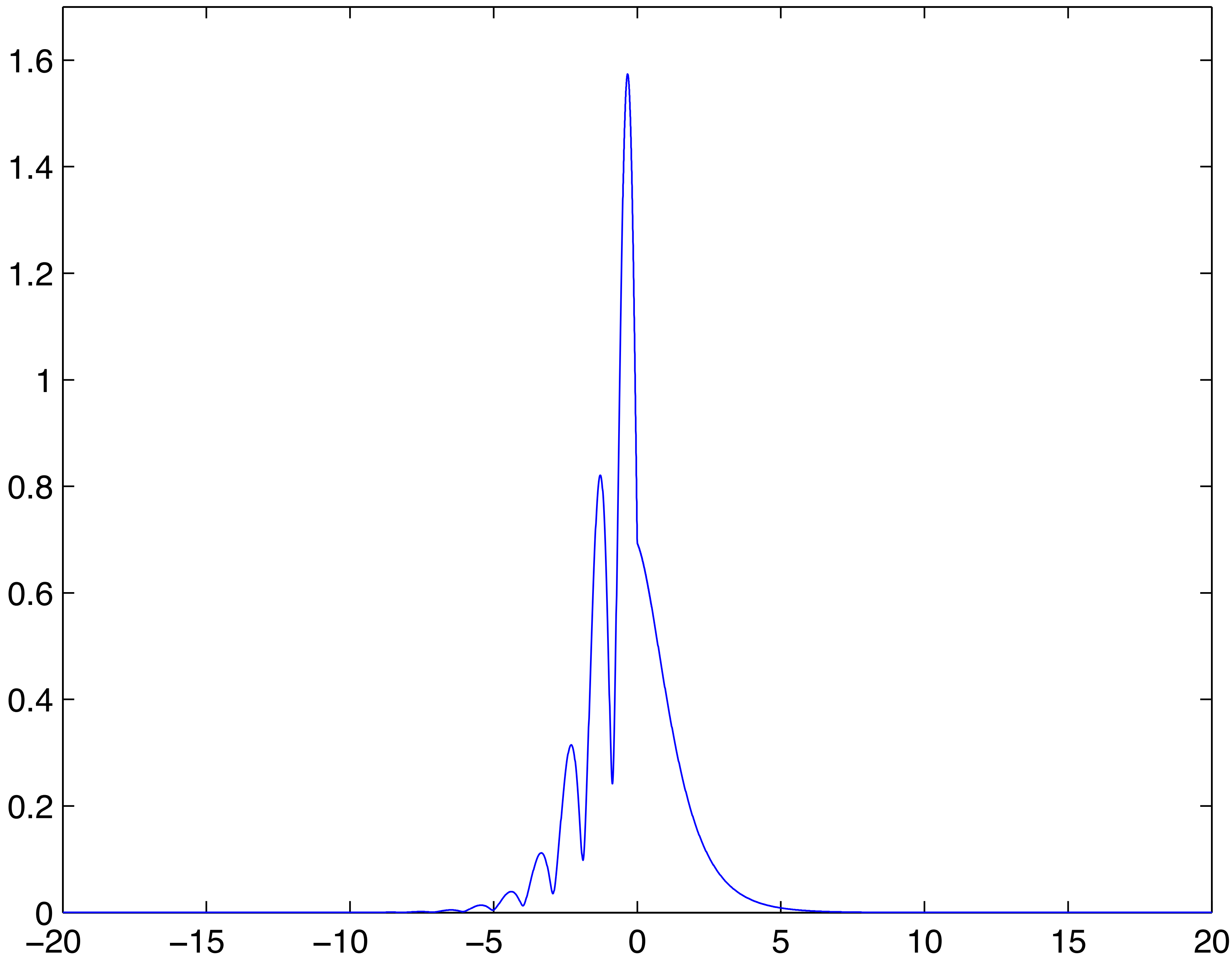

1.6
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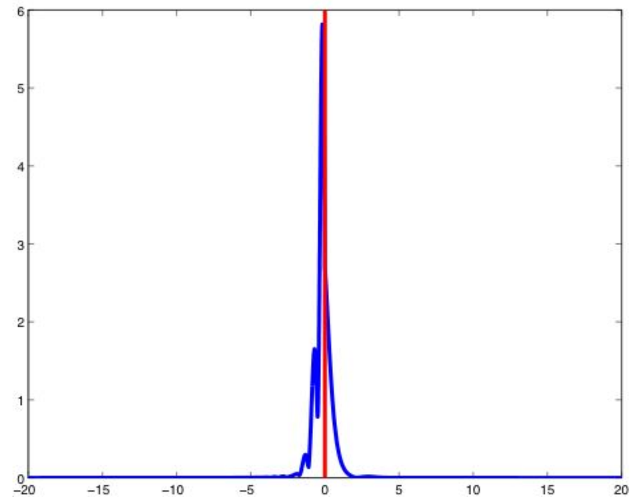
20

Phase 2 (Interaction). 

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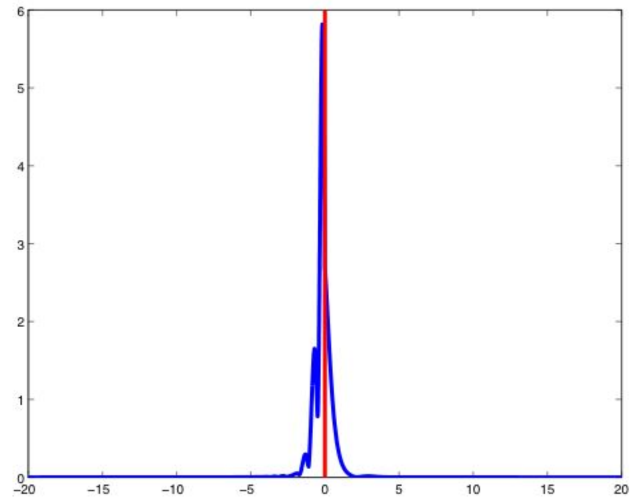

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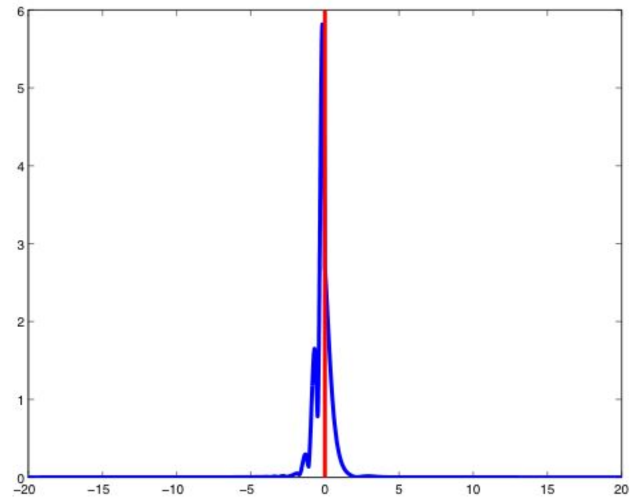

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$$u(x, t_2)$$

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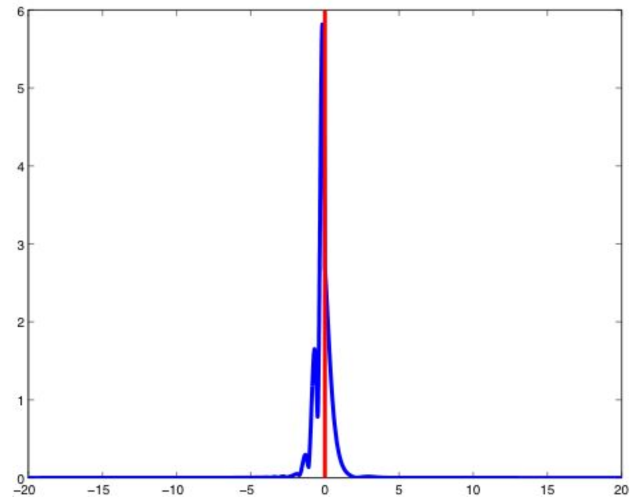
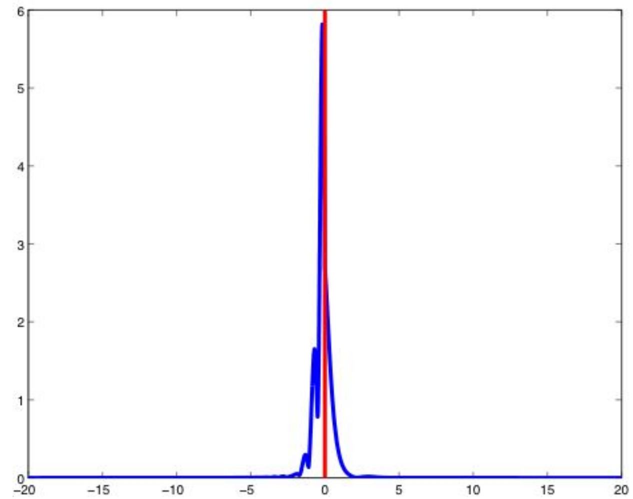
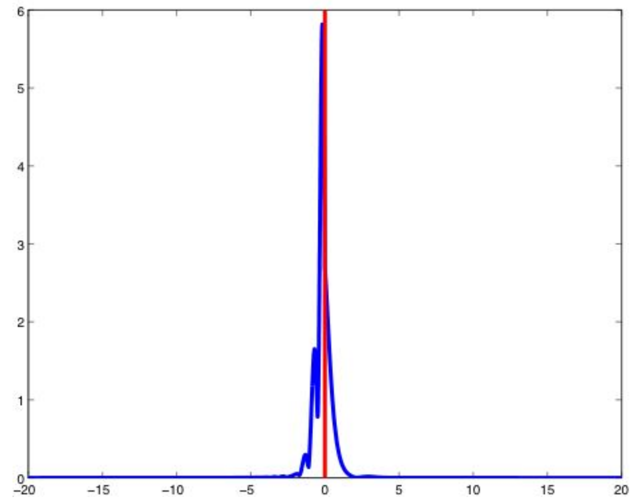


$t_1 \le t \le t_2 = t_1 + 2v^{-\delta}$,

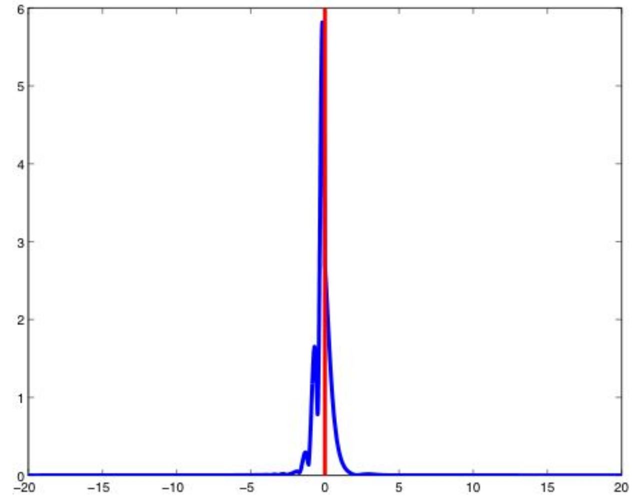
$$u(x,t_2) = t(v)e^{-it_2v^2/2}e^{it_2/2}e^{ixv}\mathsf{sech}(x - x_0 - vt_2) + r(v)e^{-it_2v^2/2}e^{it_2/2}e^{-ixv}\mathsf{sech}(x + x_0 + vt_2) + \mathcal{O}(v^{-\frac{1}{2}\delta})$$

Phase 2 (Interaction).

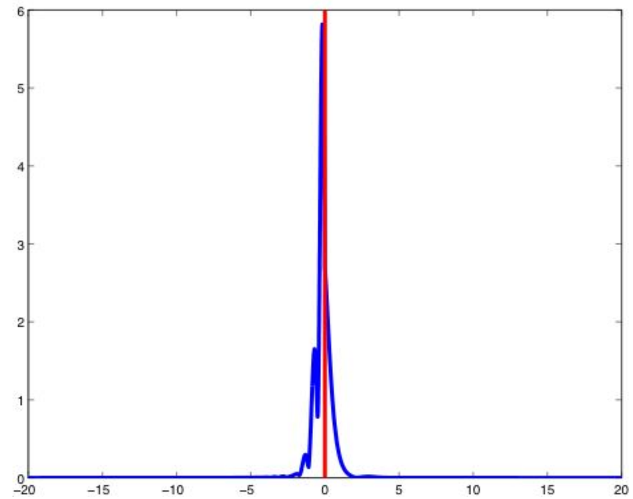


Phase 2 (Interaction). 

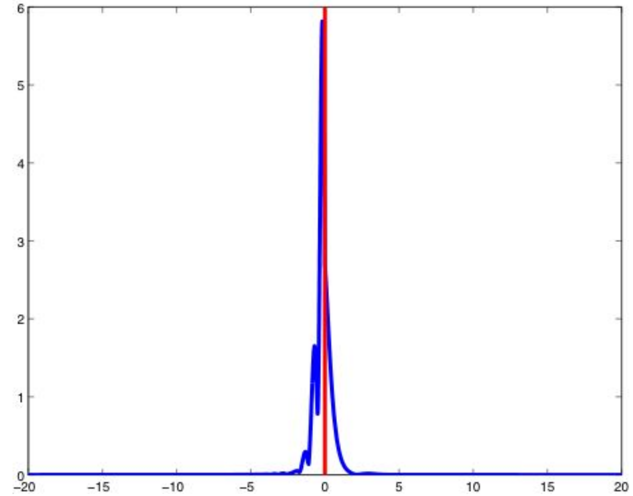
- Solution is approximated by the linear flow
-
-

Phase 2 (Interaction). 

- Solution is approximated by the linear flow
- Linear flow is approximated by the sum of
-

Phase 2 (Interaction). 

- Solution is approximated by the linear flow
- Linear flow is approximated by the sum of transmitted and reflected free linear flows
-

Phase 2 (Interaction). 

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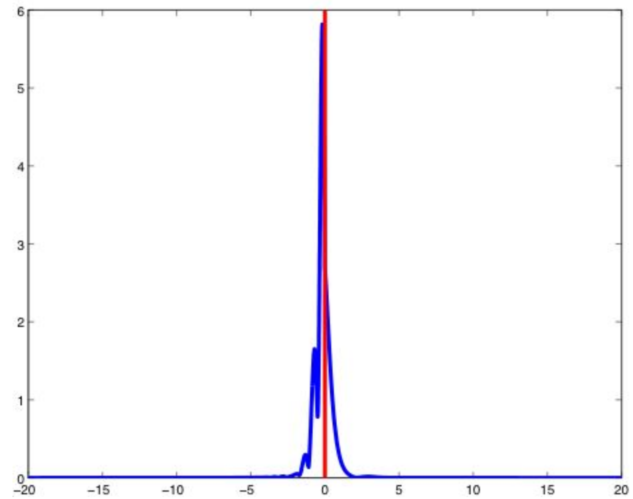
Phase 2 (Interaction). 

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Phase 3 (Post-interaction).

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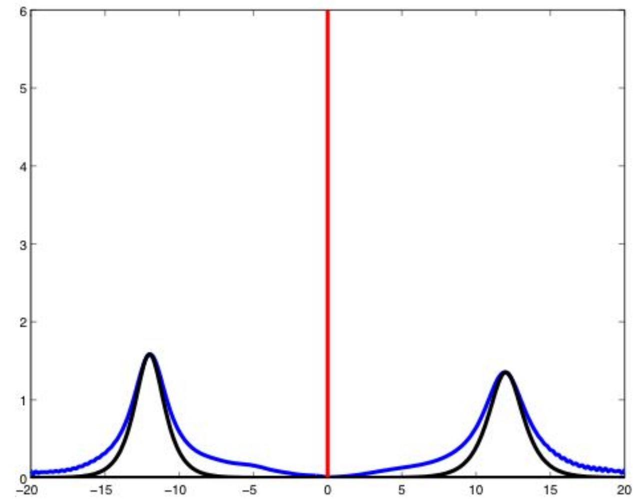

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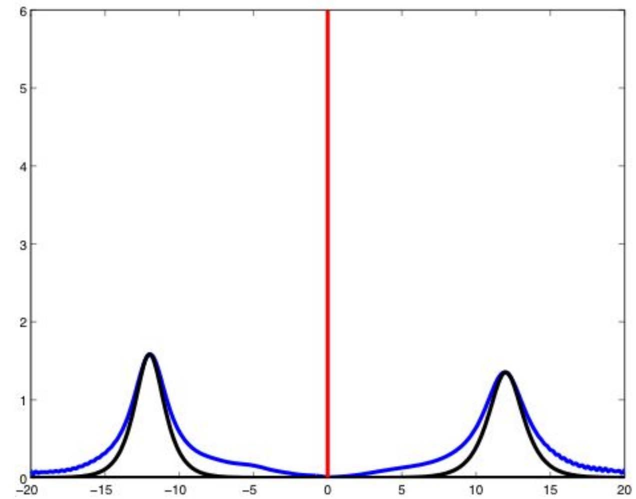
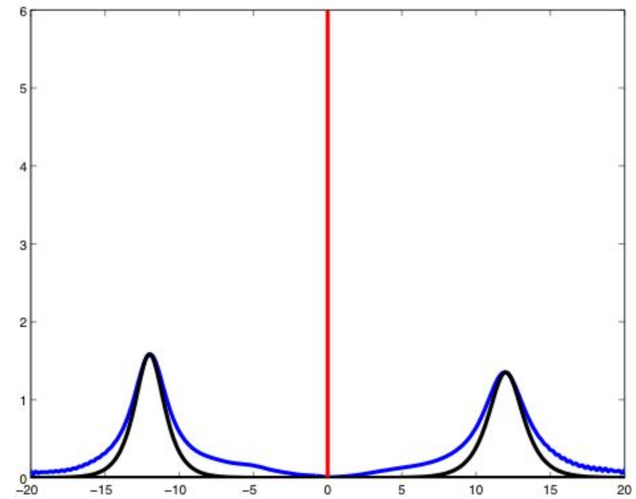


Now $t_2 < t < t_3 = t_2 + (1 - \delta)\log v$

Phase 3 (Post-interaction).



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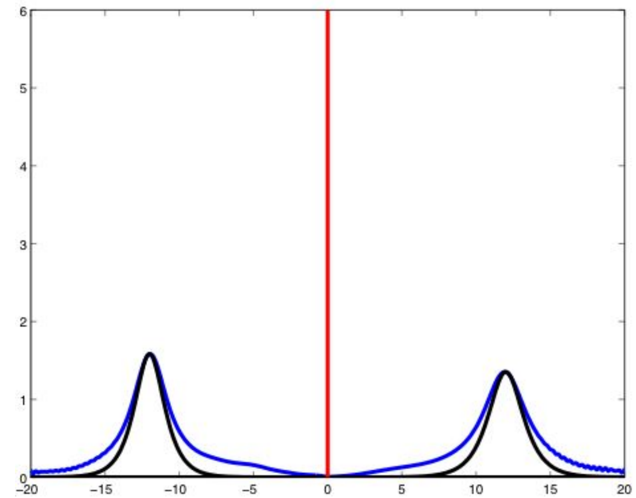
Phase 3 (Post-interaction). 

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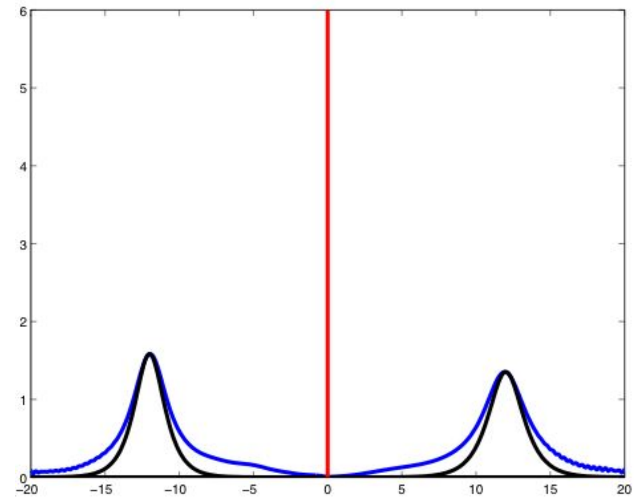
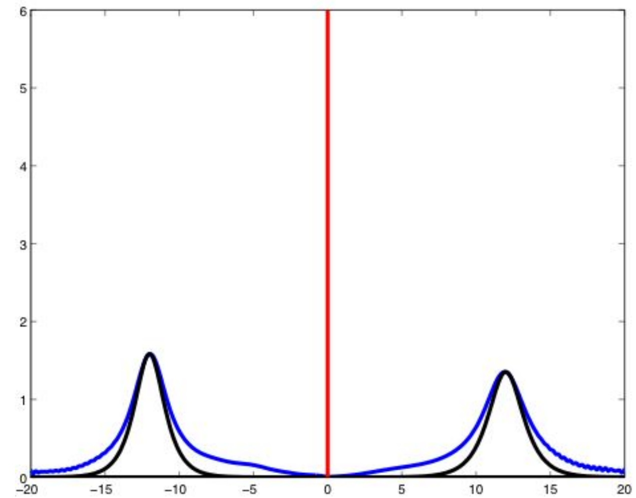
Phase 3 (Post-interaction).



Now $t_2 < t < t_3 = t_2 + (1-\delta)\log v$ and a perturbative argument enables us to evolve in time at the expense of enlarging the error

Phase 3 (Post-interaction).



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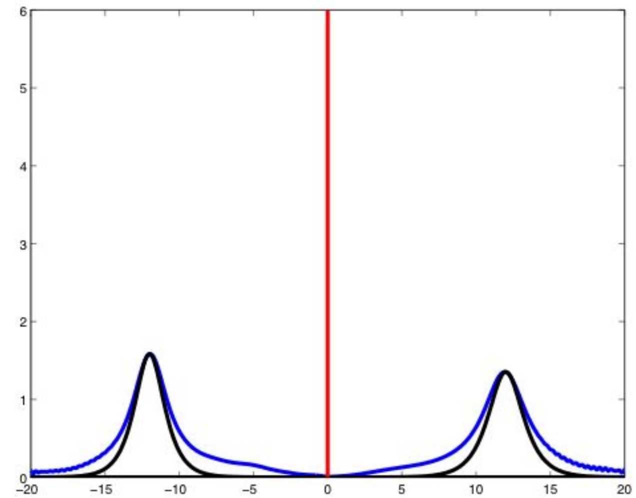
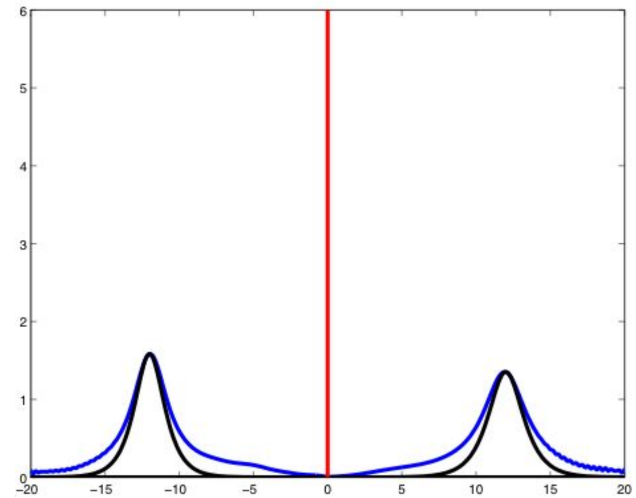
Phase 3 (Post-interaction). 

Now $t_2 < t < t_3 = t_2 + (1-\delta)\log v$ and a perturbative argument enables us to evolve in time at the expense of enlarging the error by a multiplicative factor of $e^{(1-\delta)\log v} = v^{1-\delta}$.

Phase 3 (Post-interaction). 

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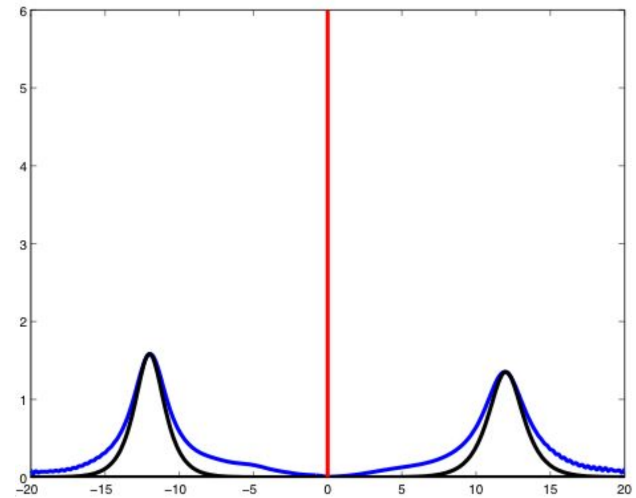

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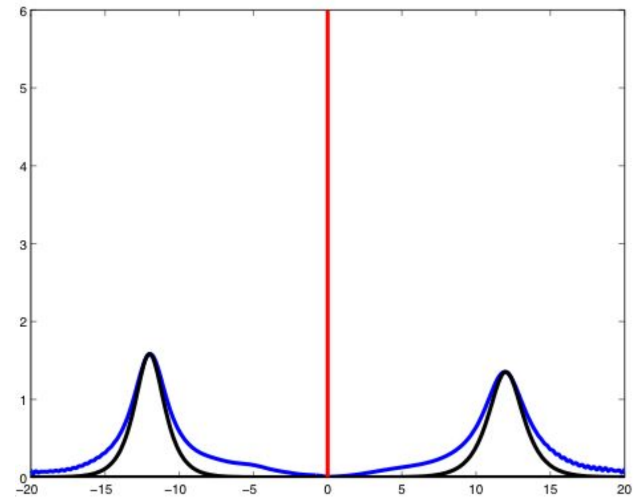
Phase 3 (Post-interaction). 

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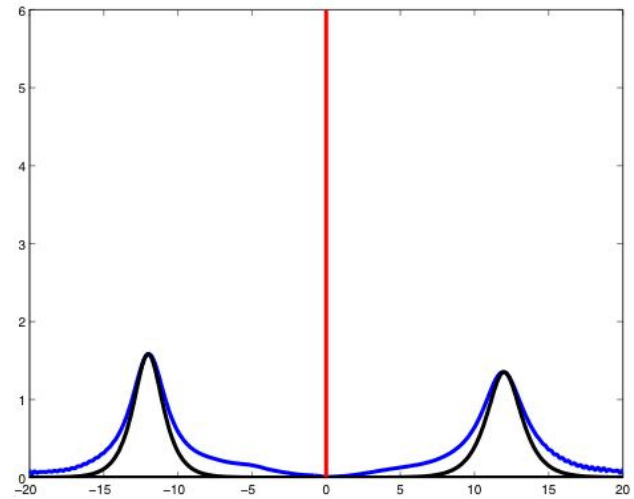
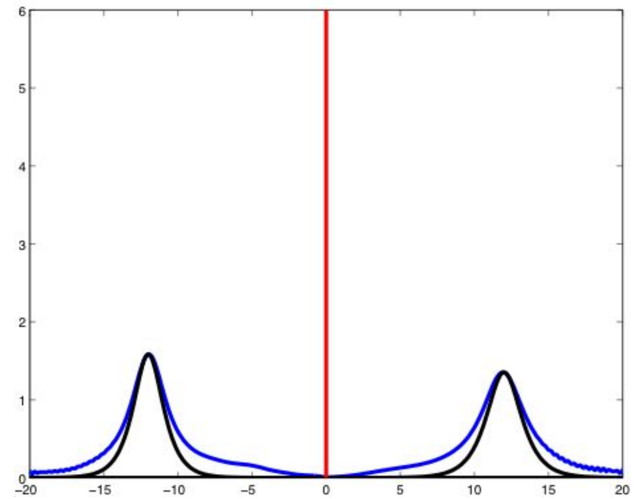
Phase 3 (Post-interaction). 

Phase 3 (Post-interaction). 

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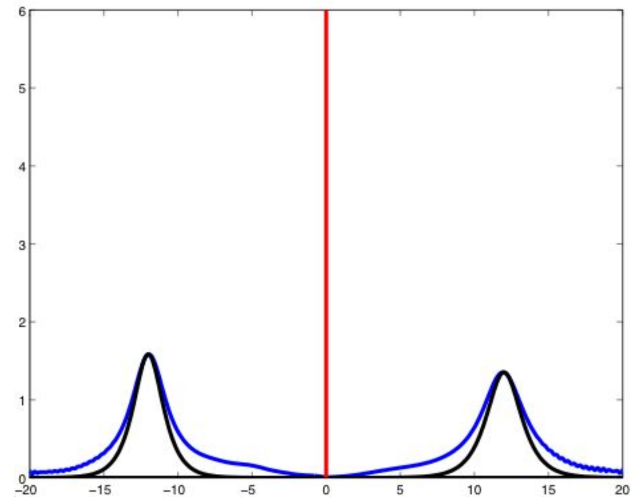
Phase 3 (Post-interaction). 

Hence $u(x,t) =$

$$e^{-itv^2/2}e^{it_2/2}e^{ixv}\mathsf{NLS}_0(t-t_2)[t(v)\mathsf{sech}(x)](x-x_0-tv)$$

$$+\, e^{-itv^2/2}e^{it_2/2}e^{-ixv}\mathsf{NLS}_0(t-t_2)[r(v)\mathsf{sech}(x)](x+x_0+tv)$$

$$+\, \mathcal{O}(v^{1-\frac{3}{2}\delta}), \qquad t_2 \le t \le t_3$$

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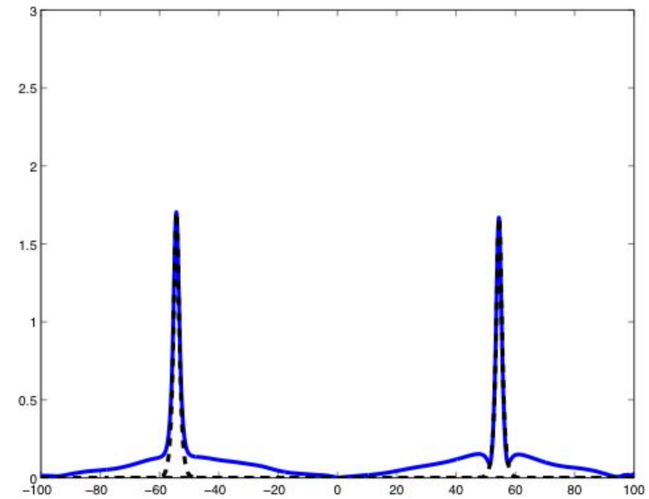

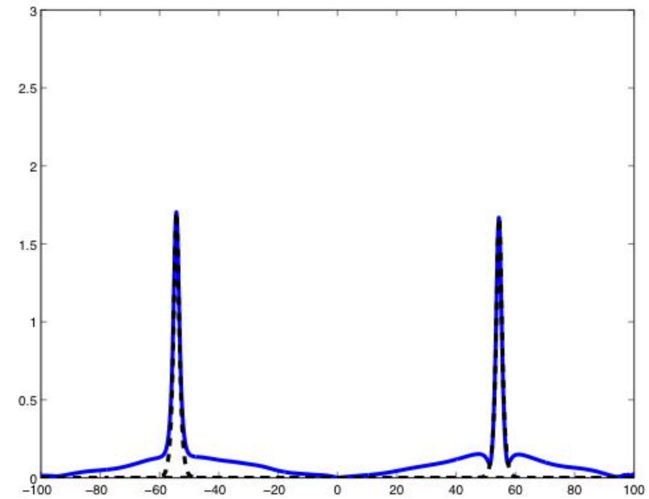

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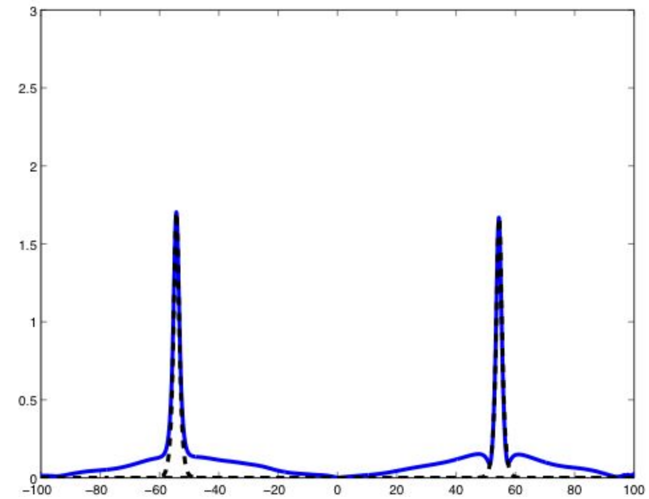

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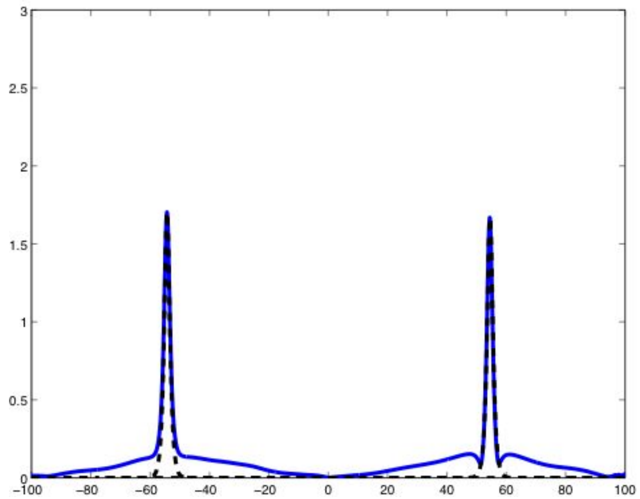

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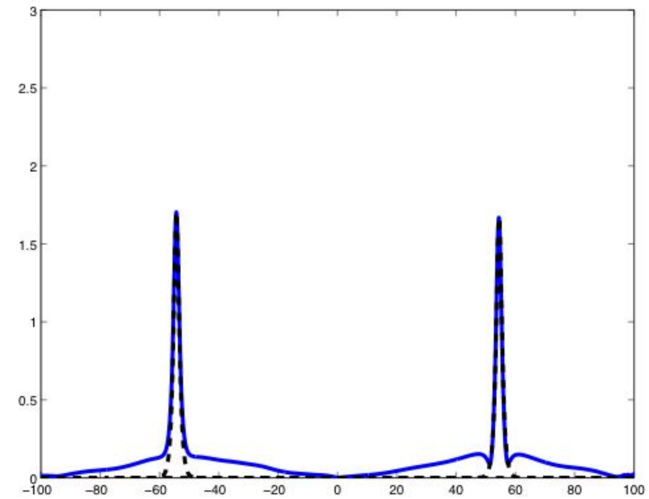

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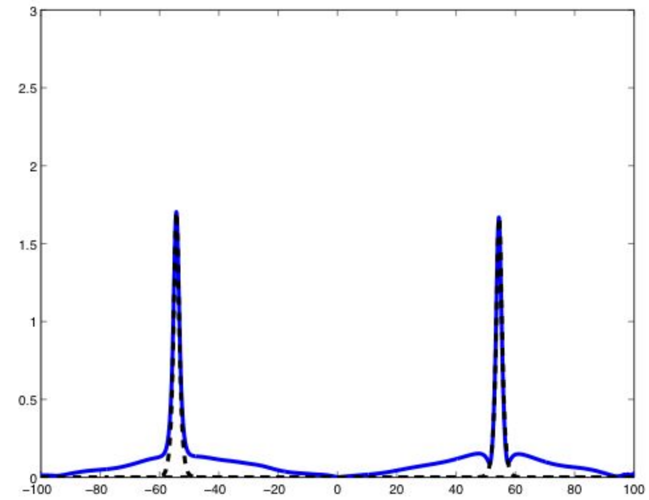

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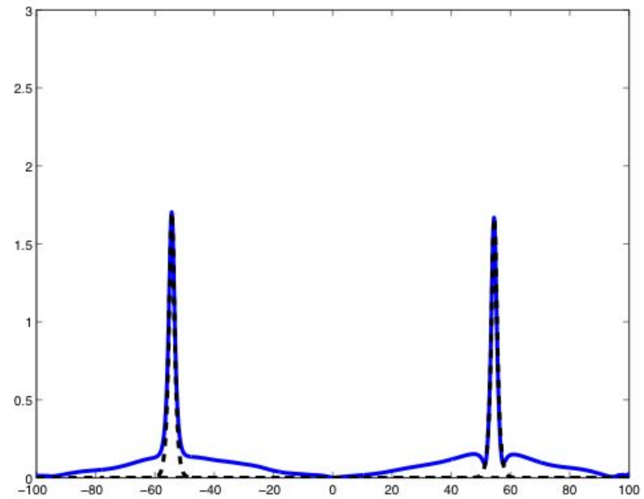

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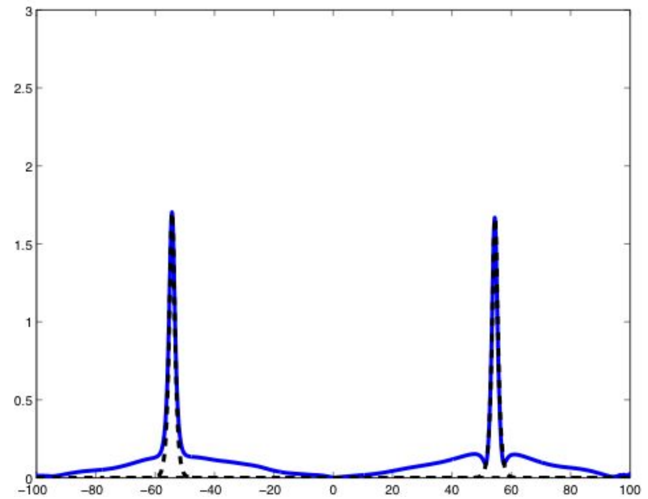

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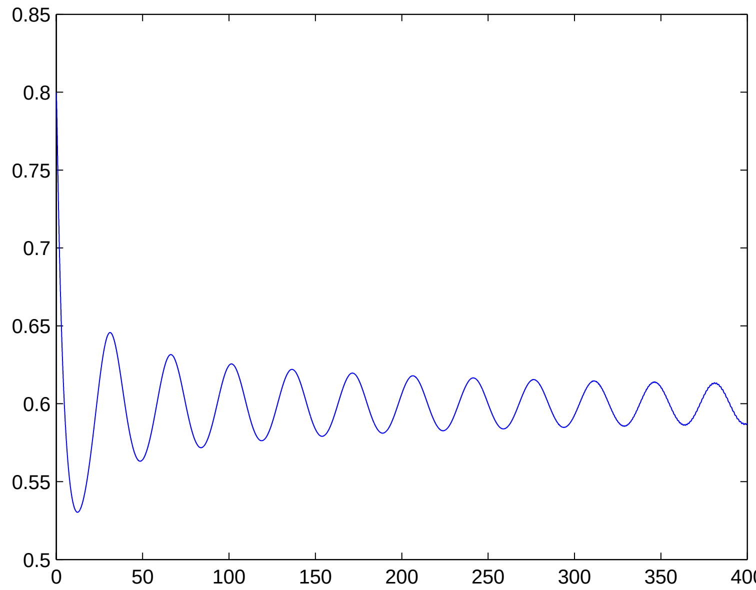

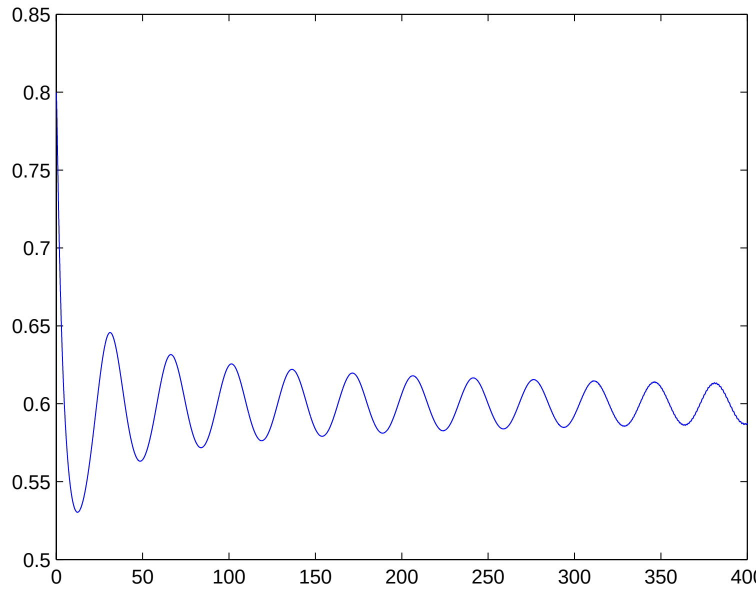

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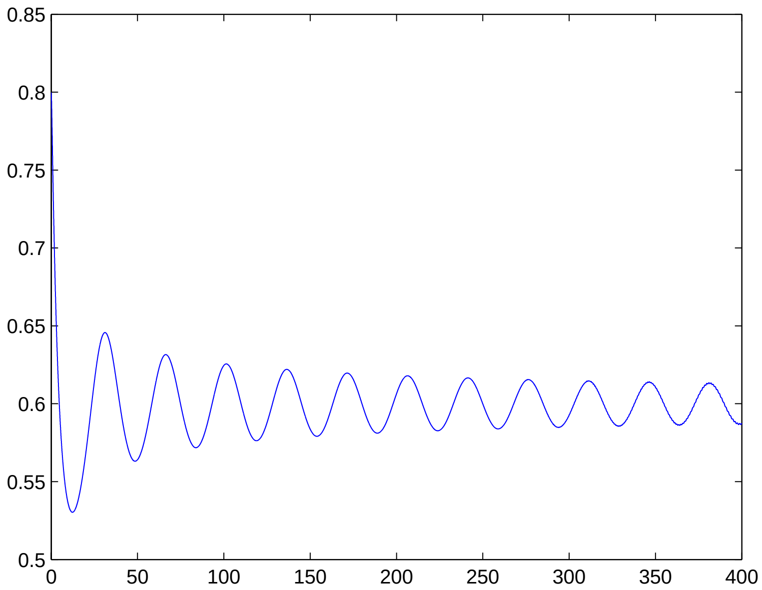

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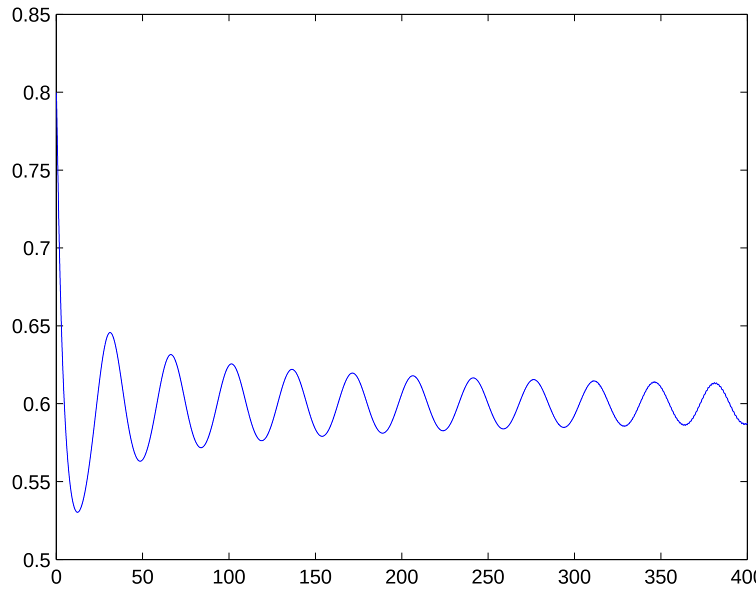

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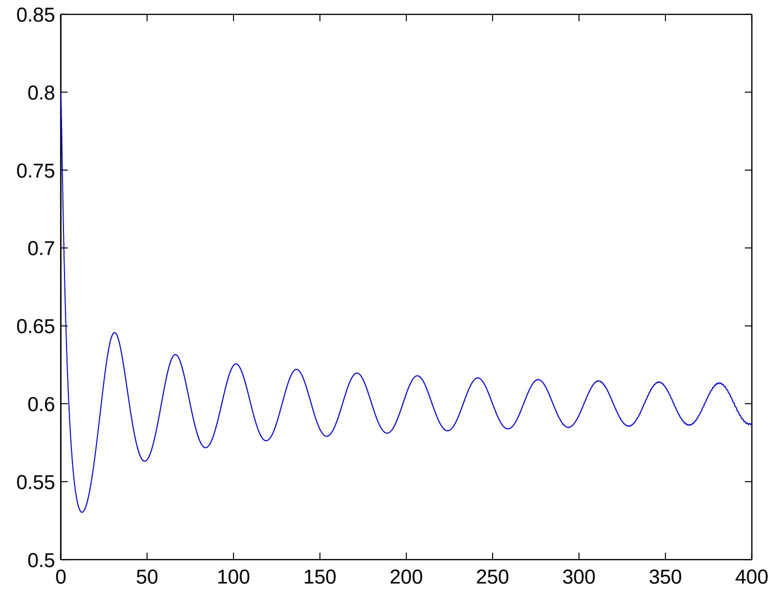

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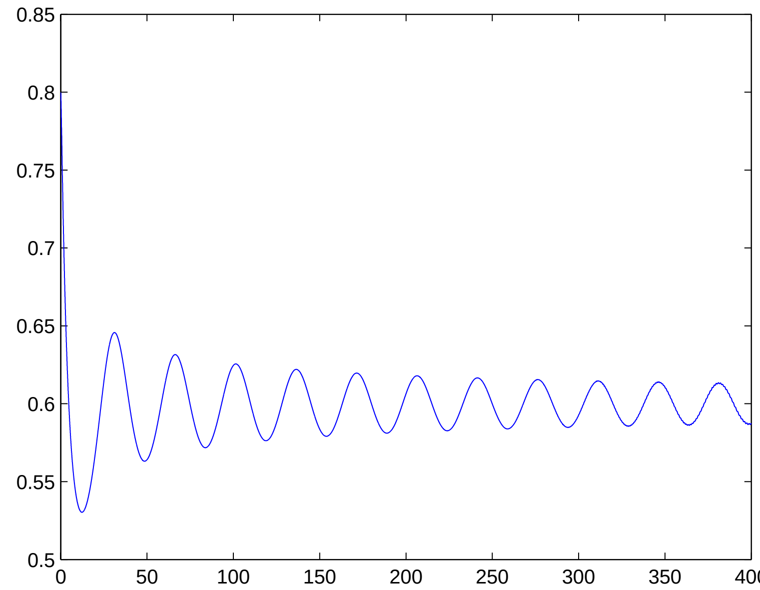

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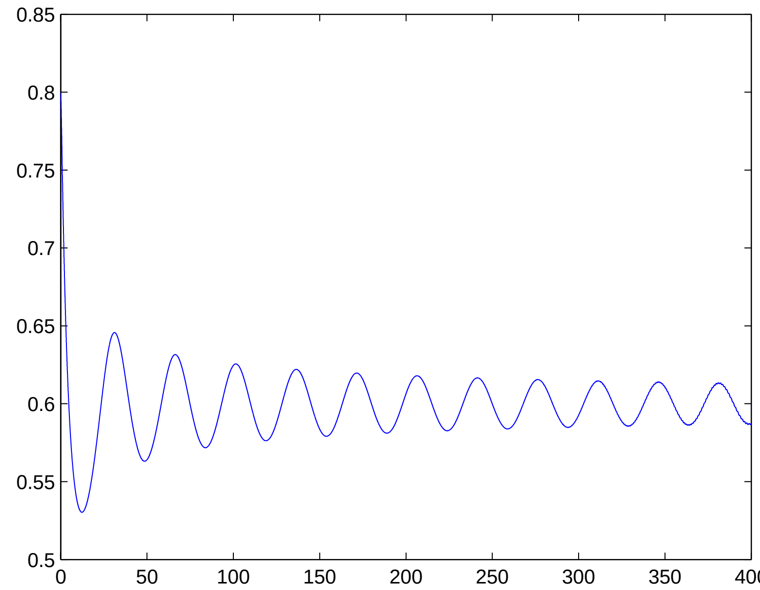

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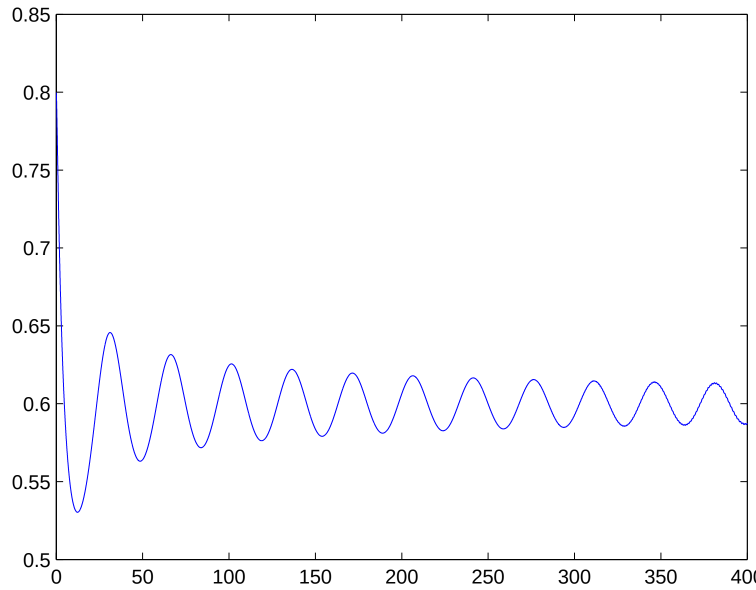

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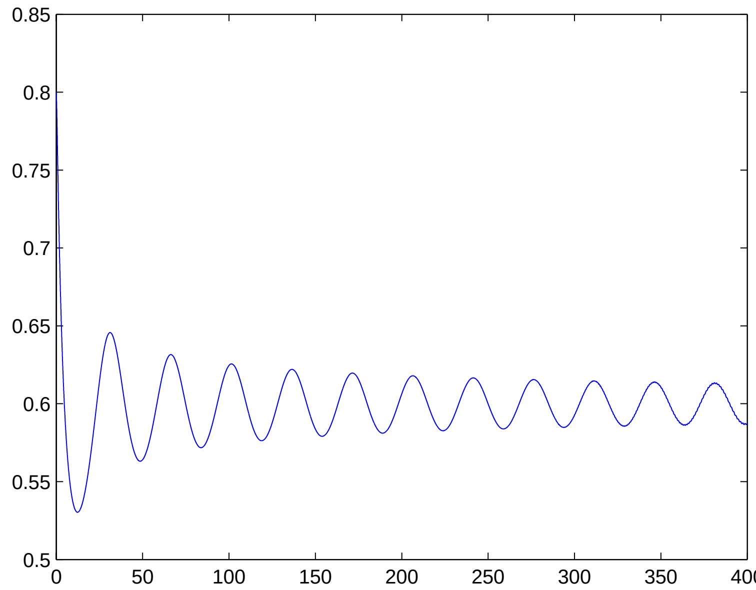

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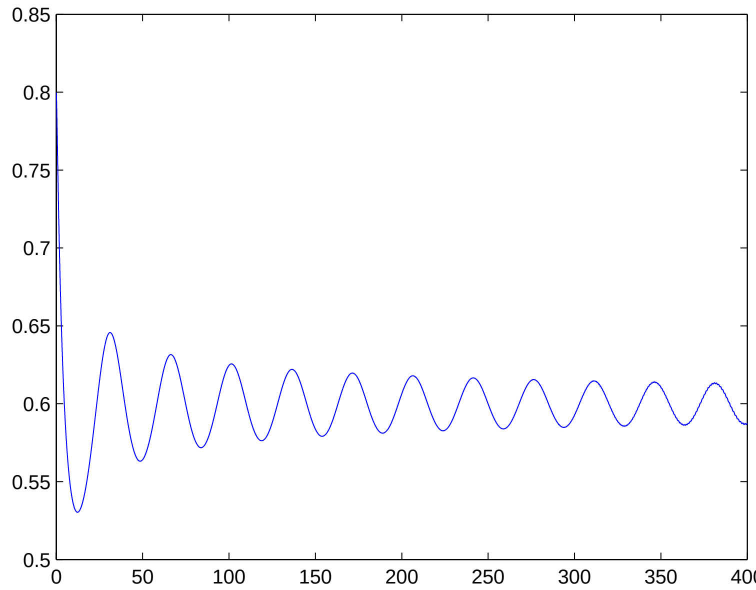

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So we end up with scattering coefficients again!

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- Similar results are true for $q < 0$ (in progress...).