MATH 1B PRACTICE FINAL EXAM

Problem 1. (9 points) Evaluate the following integrals

$$a) \quad \int (1+\sqrt{1+x})^{-1} dx$$
$$b) \quad \int x \ln \sqrt{1+x^2} dx$$
$$c) \quad \int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x dx$$

Problem 2. (12 points) Solve the following differential equations

$$a) \quad y'' - y' = e^x\,, \quad y(0) = y'(0) = 0$$
$$b) \quad y' = (1+x)/(xy)\,, \quad x > 0\,,\ y(1) = 2$$
$$c) \quad y' + xy = x\,, \quad y(0) = 1/2$$

Problem 3. (12 points) Match the *direction fields* in the figure files 1-4 to the *differential equations*:

$$a)\ y' = x(1-y^2) \qquad b)\ y' = x(1-y^3)$$
$$c)\ y' = x^5 - y^5 \qquad d)\ y' = x^2 - y^2\,.$$

Determine the *equilibrium* solutions in each case, and state if they are stable, unstable, or neither.

Please note that in the figures the coordinates (x, y) are not on the axes but on the boundaries of the figure.

Problem 4. (6 points) The radius of convergence of

$$\sum_{n=0}^{\infty} \frac{x^{4n}}{16^n}$$

is a) 0 b) 1 c) 2 d) ∞ e) none of the above?

Problem 5. (7 points) The initial value problem

$$y'' + x^2 y' = 0\,, \quad y(0) = 0\,, \quad y'(0) = 1/3\,,$$

is solved by the following power series

$$a) \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(3n+1)3^n n!} \qquad b) \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(3n+1)3^{n+1} n!}$$
$$c) \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{3n+1}}{(3n+1)3^n n!} \qquad d) \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3n!}$$

or *e*) none of the above.

Problem 6. (6 points) The series

$$\sum_{n=0}^{\infty} 2^n \sin\left(\frac{1}{3^n}\right)$$

a) diverges b) converges by the alternating series test c) converges by the root test d) converges by the comparison test e) converges by the integral test?

Problem 7. (6 points) The integrating factor for the linear first order equation

$$y' + y/x = x^2 \tag{1}$$

is a) x b) x^2 c) x^{-1} c) $\ln x$ d) none of the above?

Problem 8. (6 points) The solution of the equation (1) with $y(2) = 2$ is a) $x^4/8$ b) $x^3/4$ c) x d) $x(1 + \ln x)$ e) none of the above?

Problem 9. (6 points) The partial fraction expansion of

$$\frac{x}{(x^2 + 2x + 1)(x^2 + 1)}$$

is

$$\begin{array}{ll} a)\ \dfrac{1}{2(x+1)^2} + \dfrac{1}{2(x^2+1)} & b)\ -\dfrac{1}{2(x+1)} + \dfrac{1}{2(x^2+1)} \\ c)\ -\dfrac{1}{2(x+1)^2} + \dfrac{1}{2(x^2+1)} & d)\ -\dfrac{1}{2(x+1)} + \dfrac{x+1}{2(x^2+1)} \\ e) - \dfrac{1}{(x+1)} + \dfrac{1}{(x+1)^2} + \dfrac{x+1}{2(x^2+1)} & \end{array}$$

Problem 10. (6 points)

$$\int_0^x e^{t^2}\,dt = \qquad a)\ e^{x^2} - 1 \qquad b)\ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$c)\ \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!} \qquad d)\ \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(n+1)!} \qquad e) \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)(2n)!}$$

Problem 11. (6 points) Choose the optimal estimate among the ones given:

$$\left| \sum_{n=100}^{\infty} \frac{(-1)^n}{n^{\frac{1}{2}}} \right| \leq \begin{cases} a) & 0.01 \\ b) & 0.1 \\ c) & 1/99 \\ d) & 1/\sqrt{99} \\ e) & 1/\sqrt{101} \end{cases}$$

Problem 12. (6 points) The change in the populations of wolves, $W(t)$, and rabbits, $R(t)$, as a function of time, is modeled by the following system of equations

$$R' = \alpha R + \beta RW$$
$$W' = \gamma W + \delta RW$$

where

$$\begin{array}{ll} a)\ \alpha > 0\,,\ \beta > 0\,,\ \gamma > 0\,,\ \delta > 0 & b)\ \alpha > 0\,,\ \beta < 0\,,\ \gamma > 0\,,\ \delta > 0 \\ c)\ \alpha > 0\,,\ \beta < 0\,,\ \gamma < 0\,,\ \delta > 0 & d)\ \alpha > 0\,,\ \beta < 0\,,\ \gamma < 0\,,\ \delta < 0 \\ e)\ \alpha < 0\,,\ \beta < 0\,,\ \gamma > 0\,,\ \delta > 0\,. & \end{array}$$

(You do not have to remember this: just use your intuition!)

Problem 13. (6 points) The estimates

$$|E_1| \le \frac{K_1(b-a)^3}{12n^2}\,, \quad |E_2| \le \frac{K_2(b-a)^5}{180n^4}\,,$$

are

a) Error estimates for the Simpson (E_1) and Trapezoid (E_2) rules, and $|f^{(4)}(x)| \le K_1$, $|f''(x)| \le K_2$, for $a \le x \le b$.

b) Error estimates for the Trapezoid (E_1), and Simpson (E_2) rules, and $|f''(x)| \le K_1$, $|f^{(4)}(x)| \le K_2$, for $a \le x \le b$.

c) Error estimates for the Trapezoid (E_1), and Simpson (E_2) rules, and $|f^{(4)}(x)| \le K_1$, $|f''(x)| \le K_2$, for $a \le x \le b$.

d) Error estimates for the Simpson (E_1) and Trapezoid (E_2) rules, and $|f''(x)| \le K_1$, $|f(4)(x)| \le K_2$, for $a \le x \le b$.

e) Something else altogether.

Problem 14. (6 points) The series

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{2p}}\,, \quad p > 0\,,$$

a) converges absolutely for all $p > 0$
b) converges conditionally for all $p > 0$
c) converges for $p > 1/2$ and diverges for $p \le 1/2$
d) converges absolutely for $p > 1/2$ and converges conditionally for $p \le 1/2$
e) converges absolutely for $p > 1$ and converges conditionally for $p \le 1$.