MATH 1B PRACTICE MIDTERM # 2

Problem 1. (15 points) Determine the radius of convergence, and the convergence at the end points, of

$$\sum_{n=2}^{\infty} \frac{x^{n-1}}{(n+1)\ln n}\,.$$

Problem 2. (15 points) Find the following infinite sum (as a function of x):

$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\,.$$

Problem 3. (20 points) Show that

$$(1+x)\ln(1+x) = x + \frac{x^2}{2} - \frac{x^3}{3\cdot 2} + \frac{x^4}{4\cdot 3} - \frac{x^5}{5\cdot 4} + \cdots$$

Hint: Differentiate both sides with respect to x.

Problem 4. (20 points) Find the 4th Taylor polynomial of $\tan x$ at $x = 0$ and clearly state the formula for the remainder estimate valid for $|x| \le \pi/4$ (without calculating the numbers!).

Problem 5. (15 points) Sketch the *direction field* of

$$y' = y(1-y)\,,$$

and determine the equilibrium solutions. Are they stable?

Problem 6. (15 points) Solve the equation

$$y' = \frac{e^x}{4y^3}\,, \quad y(0) = 1\,.$$