MATH 1B PRACTICE MIDTERM # 2

Problem 1. Determine the radii of convergence of

$$a) \quad \sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

$$b) \quad \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n+1)\ln^2 n}$$

Do the series converge at the end points?

Problem 2. Find the power series expansions (centered at 0) of the following functions

$$a) \quad f(x) = x(4x^2+1)^{-1}$$

$$b) \quad f(x) = \sin 2x + x\cos 3x$$

Where are the expansions valid (i.e. for what values of x do they converge)?

Problem 3. Find the the third order Taylor polynomial of $1/\sin x$ at $\pi/2$.

Problem 4. Write the integral

$$\int_0^1 e^{x^3}\,dx$$

as an infinite series.

Problem 5. Consider the 5th order Taylor polynomial of $\sin x$ at π. For $|x-\pi| \leq \pi$ determine which gives a more accurate measure of the quality of the approximation: the Taylor remainder estimate *or* the alternating series error bound.

Problem 6. Sketch the *direction field* of

$$y' = \frac{x^2}{x^2+1}(y^2-1)\,,$$

and determine the equilibrium solutions. Are they stable?

Problem 7. Solve the equation

$$y' = 2 + 2y + x + xy\,.$$