MATH 1B SECOND PRACTICE MIDTERM # 1

**On the exam you will be asked to solve each problem on a separate
sheet writing your name and section number on each sheet.**

Problem 1.

a) Evaluate the following (indefinite) integral

$$\int (1 + x^{1/3})^{-1} dx$$

b) Evaluate the following (indefinite) integral

$$\int x^3 \cos(x^2) dx$$

Problem 2. Evaluate the following improper integral (and state why this is an
improper integral):

$$\int_1^9 (x - 9)^{-1/3} dx$$

Problem 3. Determine (providing an explanation) convergence or divergence of
the following series:

$$a) \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1) \ln^2 n}$$

$$b) \sum_{n=2}^{\infty} \frac{4^n n!}{5 \cdot 8 \cdot 11 \cdots (3n + 2)}$$

Problem 4. Explain why the following infinite series converges:

$$\sum_{n=1}^{\infty} (-1)^n \sin(1/n),$$

and give an *error estimation* in the approximation using 1000 term in the series
(you can use the fact that $\sin x \le x$, $x \ge 0$).

Problem 5. Let E_S be the error bound for the Simpson's rule. The estimate on
the error says that if

$$|f^{(p)}(x)| \le K \quad \text{for } a \le x \le b$$

then

$$|E_S| \le \frac{K(a - b)^5}{180 n^l}.$$

What are p and l?