

2. (a)  $g(x) = \int_0^x f(t) dt$ , so  $g(0) = \int_0^0 f(t) dt = 0.$

$$g(1) = \int_0^1 f(t) dt = \frac{1}{2} \cdot 1 \cdot 1 \text{ [area of triangle]} = \frac{1}{2}.$$

$$\begin{aligned} g(2) &= \int_0^2 f(t) dt = \int_0^1 f(t) dt + \int_1^2 f(t) dt \text{ [below the } x\text{-axis]} \\ &= \frac{1}{2} - \frac{1}{2} \cdot 1 \cdot 1 = 0. \end{aligned}$$

$$g(3) = g(2) + \int_2^3 f(t) dt = 0 - \frac{1}{2} \cdot 1 \cdot 1 = -\frac{1}{2}.$$

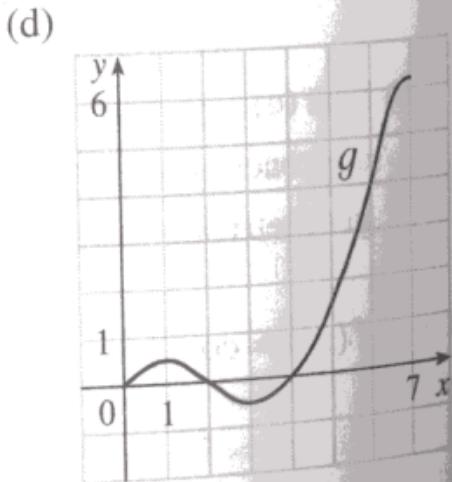
$$g(4) = g(3) + \int_3^4 f(t) dt = -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = 0.$$

$$g(5) = g(4) + \int_4^5 f(t) dt = 0 + 1.5 = 1.5.$$

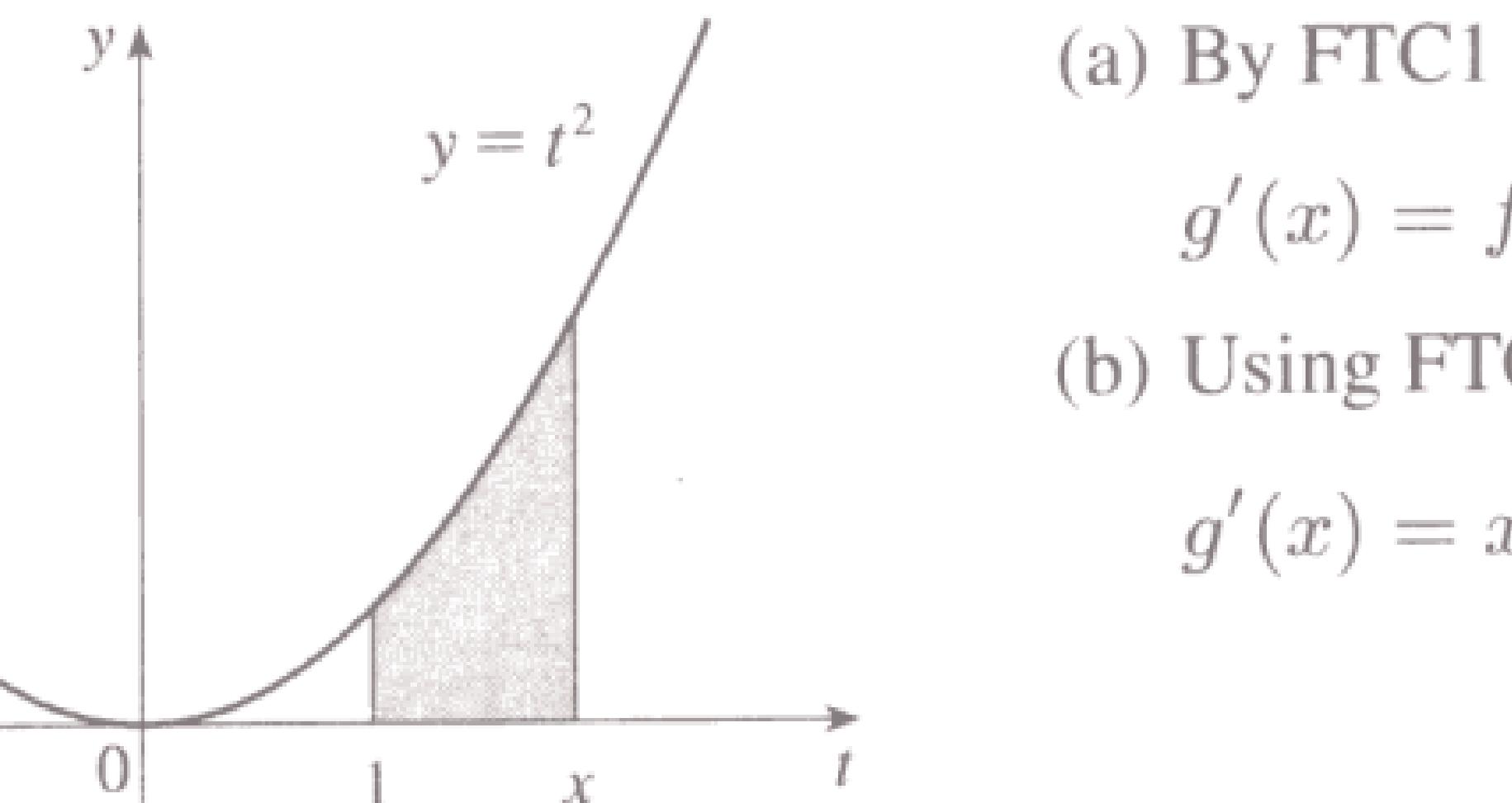
$$g(6) = g(5) + \int_5^6 f(t) dt = 1.5 + 2.5 = 4.$$

(b)  $g(7) = g(6) + \int_6^7 f(t) dt \approx 4 + 2.2 \text{ [estimate from the graph]} = 6.2.$

(c) The answers from part (a) and part (b) indicate that  $g$  has a minimum at  $x = 3$  and a maximum at  $x = 7$ . This makes sense from the graph of  $f$  since we are subtracting area on  $1 < x < 3$  and adding area on  $3 < x < 7$ .



5.



(a) By FTC1 with  $f(t) = t^2$  and  $a = 1$ ,  $g(x) = \int_1^x t^2 dt \Rightarrow$

$$g'(x) = f(x) = x^2.$$

(b) Using FTC2,  $g(x) = \int_1^x t^2 dt = \left[ \frac{1}{3}t^3 \right]_1^x = \frac{1}{3}x^3 - \frac{1}{3} \Rightarrow$

$$g'(x) = x^2.$$

9.  $f(t) = t^2 \sin t$  and  $g(y) = \int_2^y t^2 \sin t dt$ , so by FTC1,  $g'(y) = y^2 \sin y$ .

10.  $f(x) = \frac{1}{x+x^2}$  and  $g(u) = \int_3^u \frac{1}{x+x^2} dx$ , so  $g'(u) = f(u) = \frac{1}{u+u^2}$ .

$$\begin{aligned} 4. \frac{d}{dx} \left[ -\frac{\sqrt{x^2 + a^2}}{a^2 x} + C \right] &= -\frac{1}{a^2} \frac{d}{dx} \left[ \frac{\sqrt{x^2 + a^2}}{x} \right] = -\frac{x(x/\sqrt{x^2 + a^2}) - \sqrt{x^2 + a^2} \cdot 1}{a^2 x^2} \\ &= -\frac{x^2 - (x^2 + a^2)}{a^2 x^2 \sqrt{x^2 + a^2}} = \frac{1}{x^2 \sqrt{x^2 + a^2}} \end{aligned}$$

$$-5\left(0 - \frac{\pi}{2}\right) - (2 - 0) = -\frac{5\pi}{2} = -3.5$$

$$\begin{aligned} 40. \int_0^{3\pi/2} |\sin x| \, dx &= \int_0^{\pi} \sin x \, dx + \int_{\pi}^{3\pi/2} (-\sin x) \, dx = [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{3\pi/2} \\ &= [1 - (-1)] + [0 - (-1)] = 2 + 1 = 3 \end{aligned}$$

$$v(0) = C = -4 \Rightarrow v(t) = t^2 + 3t - 4$$

56. (a)  $v'(t) = a(t) = 2t + 3 \Rightarrow v(t) = t^2 + 3t + C \Rightarrow v(0) = C = -4 \Rightarrow v(t) = t^2 + 3t - 4$

(b) distance traveled  $= \int_0^3 |t^2 + 3t - 4| dt = \int_0^3 |(t+4)(t-1)| dt$

$$\begin{aligned} &= \int_0^1 (-t^2 - 3t + 4) dt + \int_1^3 (t^2 + 3t - 4) dt \\ &= \left[ -\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t \right]_0^1 + \left[ \frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t \right]_1^3 \\ &= \left( -\frac{1}{3} - \frac{3}{2} + 4 \right) + \left( 9 + \frac{27}{2} - 12 \right) - \left( \frac{1}{3} + \frac{3}{2} - 4 \right) = \frac{89}{6} \text{ m} \end{aligned}$$

4. Let  $u = x$ ,  $dv = e^{-x} dx \Rightarrow du = dx$ ,  $v = -e^{-x}$ . Then

$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + C.$$

10. Let  $u = \sin^{-1} x$ ,  $du = dx \rightarrow dx = \frac{du}{\sqrt{1-x^2}}$ ,  $v = x$ . Then

$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx$ . Setting  $t = 1-x^2$ , we get  $dt = -2x \, dx$ , so

$$-\int \frac{x \, dx}{\sqrt{1-x^2}} = -\int t^{-1/2} (-\frac{1}{2} dt) = \frac{1}{2} (2t^{1/2}) + C = t^{1/2} + C = \sqrt{1-x^2} + C.$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C.$$

**30.** Let  $u = r^2$ ,  $dv = \frac{r}{\sqrt{4+r^2}} dr \Rightarrow du = 2r dr$ ,  $v = \sqrt{4+r^2}$ . By (6),

$$\begin{aligned} \int_0^1 \frac{r^3}{\sqrt{4+r^2}} dr &= \left[ r^2 \sqrt{4+r^2} \right]_0^1 - 2 \int_0^1 r \sqrt{4+r^2} dr = \sqrt{5} - \frac{2}{3} \left[ (4+r^2)^{3/2} \right]_0^1 \\ &= \sqrt{5} - \frac{2}{3}(5)^{3/2} + \frac{2}{3}(8) = \sqrt{5} \left( 1 - \frac{10}{3} \right) + \frac{16}{3} = \frac{16}{3} - \frac{7}{3}\sqrt{5} \end{aligned}$$

**45.** Let  $u = (\ln x)^n$ ,  $dv = dx \Rightarrow du = n(\ln x)^{n-1}(dx/x)$ ,  $v = x$ . By Equation 2,

$$\int (\ln x)^n dx = x(\ln x)^n - \int nx(\ln x)^{n-1}(dx/x) = x(\ln x)^n - n \int (\ln x)^{n-1} dx.$$

**46.** Let  $u = x^n$ ,  $dv = e^x dx \Rightarrow du = nx^{n-1} dx$ ,  $v = e^x$ . By Equation 2,  $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$ .

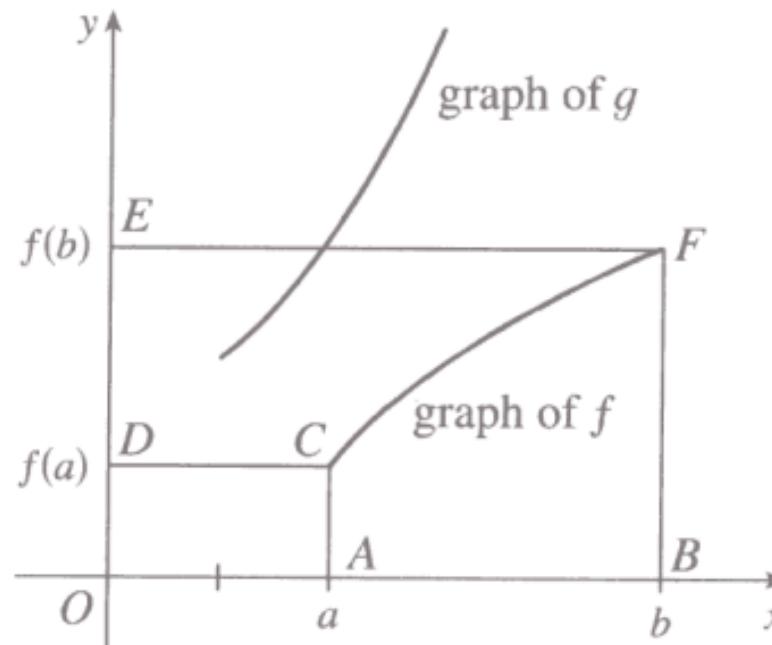
64. (a) Take  $g(x) = x$  and  $g'(x) = 1$  in Equation 1.

(b) By part (a),  $\int_a^b f(x) dx = b f(b) - a f(a) = \int_a^b x f'(x) dx$ . Now let  $y = f(x)$ , so that  $x = g(y)$  and  $dy = f'(x) dx$ . Then  $\int_a^b x f'(x) dx = \int_{f(a)}^{f(b)} g(y) dy$ . The result follows.

(c) Part (b) says that the area of region  $ABFC$  is

$$= bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y) dy$$

$$= (\text{area of rectangle } OBFE) - (\text{area of rectangle } OACD) - (\text{area of region } DCFE)$$



(d) We have  $f(x) = \ln x$ , so  $f^{-1}(x) = e^x$ , and since  $g = f^{-1}$ , we have  $g(y) = e^y$ . By part (b),

$$\int_1^e \ln x \, dx = e \ln e - 1 \ln 1 - \int_{\ln 1}^{\ln e} e^y \, dy = e - [e^y]_0^1 = e - (e - 1) = 1.$$

$$\begin{aligned} 2 \int \sin^6 x \cos^3 x \, dx &= \int \sin^6 x \cos^2 x \cos x \, dx = \int \sin^6 x (1 - \sin^2 x) \cos x \, dx = \int u^6 (1 - u^2) \, du \\ &= \int (u^6 - u^8) \, du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C \end{aligned}$$

$$6. \int \sin^3(mx) dx = \int (1 - \cos^2 mx) \sin mx dx = -\frac{1}{m} \int (1 - u^2) du \quad [u = \cos mx, du = -m \sin mx dx]$$

$$= -\frac{1}{m} \left( u - \frac{1}{3} u^3 \right) + C = -\frac{1}{m} \left( \cos mx - \frac{1}{3} \cos^3 mx \right) + C$$

$$= \frac{1}{3m} \cos^3 mx - \frac{1}{m} \cos mx + C$$

$$23. \int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C$$

$$24. \int \tan^4 x \, dx = \int \tan^2 x (\sec^2 x - 1) \, dx = \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx = \frac{1}{3} \tan^3 x - \tan x$$

(Set  $u = \tan x$  in the first integral and use Exercise 23 for the second.)

$$\begin{aligned}
 27. \int_0^{\pi/3} \tan^5 x \sec^4 x \, dx &= \int_0^{\pi/3} \tan^5 x (\tan^2 x + 1) \sec^2 x \, dx \\
 &= \int_0^{\sqrt{3}} u^5 (u^2 + 1) \, du \quad [u = \tan x, du = \sec^2 x \, dx] \\
 &= \int_0^{\sqrt{3}} (u^7 + u^5) \, du = \left[ \frac{1}{8}u^8 + \frac{1}{6}u^6 \right]_0^{\sqrt{3}} = \frac{81}{8} + \frac{27}{6} - \frac{81}{8} + \frac{9}{2} = \frac{81}{8} + \frac{36}{8} = \frac{117}{8}
 \end{aligned}$$

$$46. \int \frac{dx}{\cos x - 1} = \int \frac{1}{\cos x - 1} \cdot \frac{\cos x + 1}{\cos x + 1} dx = \int \frac{\cos x + 1}{\cos^2 x - 1} dx$$
$$= \int \left( -\cot x \csc x - \csc^2 x \right) dx = \csc x + \cot x + C$$

**48.** Let  $u = \tan^7 x$ ,  $dv = \sec x \tan x dx \Rightarrow du = 7 \tan^6 x \sec^2 x dx$ ,  $v = \sec x$ . Then

$$\begin{aligned}\int \tan^8 x \sec x dx &= \int \tan^7 x \cdot \sec x \tan x dx = \tan^7 x \sec x - \int 7 \tan^6 x \sec^2 x \sec x dx \\&= \tan^7 x \sec x - 7 \int \tan^6 x (\tan^2 x + 1) \sec x dx \\&= \tan^7 x \sec x - 7 \int \tan^8 x \sec x dx - 7 \int \tan^6 x \sec x dx.\end{aligned}$$

Thus,  $8 \int \tan^8 x \sec x dx = \tan^7 x \sec x - 7 \int \tan^6 x \sec x dx$  and

$$\int_0^{\pi/4} \tan^8 x \sec x dx = \frac{1}{8} [\tan^7 x \sec x]_0^{\pi/4} - \frac{7}{8} \int_0^{\pi/4} \tan^6 x \sec x dx = \frac{\sqrt{2}}{8} - \frac{7}{8} I.$$

**55.** For  $0 < x < \frac{\pi}{2}$ , we have  $0 < \sin x < 1$ , so  $\sin^3 x < \sin x$ . Hence the area is

$$\int_0^{\pi/2} (\sin x - \sin^3 x) dx = \int_0^{\pi/2} \sin x (1 - \sin^2 x) dx = \int_0^{\pi/2} \cos^2 x \sin x dx. \text{ Now let } u = \cos x \Rightarrow$$

$$du = -\sin x dx. \text{ Then area} = \int_1^0 u^2 (-du) = \int_0^1 u^2 du = \left[ \frac{1}{3} u^3 \right]_0^1 = \frac{1}{3}.$$