

3. The area under the graph of  $y = 1/x^3 = x^{-3}$  between  $x = 1$  and  $x = t$  is  $A(t) = \int_1^t x^{-3} dx = [-\frac{1}{2}x^{-2}]_1^t = -\frac{1}{2}t^{-2} - (-\frac{1}{2}) = \frac{1}{2} - 1/(2t^2)$ . So the area for  $1 \leq x \leq 10$  is  $A(10) = 0.5 - 0.005 = 0.495$ , the area for  $1 \leq x \leq 100$  is  $A(100) = 0.5 - 0.00005 = 0.49995$ , and the area for  $1 \leq x \leq 1000$  is  $A(1000) = 0.5 - 0.000005 = 0.499995$ . The total area under the curve for  $x \geq 1$  is  $\lim_{t \rightarrow \infty} A(t) = \lim_{t \rightarrow \infty} [\frac{1}{2} - 1/(2t^2)] = \frac{1}{2}$ .

11.  $\int_{-\infty}^{\infty} \frac{x dx}{1+x^2} = \int_{-\infty}^0 \frac{x dx}{1+x^2} + \int_0^{\infty} \frac{x dx}{1+x^2}$  and  
 $\int_{-\infty}^0 \frac{x dx}{1+x^2} = \lim_{t \rightarrow -\infty} [\frac{1}{2} \ln(1+x^2)]_t^0 = \lim_{t \rightarrow -\infty} [0 - \frac{1}{2} \ln(1+t^2)] = -\infty$ . Divergent

13.  $\int_{-\infty}^{\infty} xe^{-x^2} dx = \int_{-\infty}^0 xe^{-x^2} dx + \int_0^{\infty} xe^{-x^2} dx$ .  
 $\int_{-\infty}^0 xe^{-x^2} dx = \lim_{t \rightarrow -\infty} (-\frac{1}{2}) [e^{-x^2}]_t^0 = \lim_{t \rightarrow -\infty} (-\frac{1}{2})(1 - e^{-t^2}) = -\frac{1}{2} \cdot 1 = -\frac{1}{2}$ , and  
 $\int_0^{\infty} xe^{-x^2} dx = \lim_{t \rightarrow \infty} (-\frac{1}{2}) [e^{-x^2}]_0^t = \lim_{t \rightarrow \infty} (-\frac{1}{2})(e^{-t^2} - 1) = -\frac{1}{2} \cdot (-1) = \frac{1}{2}$ .  
Therefore,  $\int_{-\infty}^{\infty} xe^{-x^2} dx = -\frac{1}{2} + \frac{1}{2} = 0$ . Convergent

21.  $\int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \rightarrow \infty} \left[ \frac{(\ln x)^2}{2} \right]_1^t$  (by substitution with  $u = \ln x$ ,  $du = dx/x$ ) =  $\lim_{t \rightarrow \infty} \frac{(\ln t)^2}{2} = \infty$ . Divergent

55.  $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \lim_{t \rightarrow 0^+} \int_t^1 \frac{dx}{\sqrt{x}(1+x)} + \lim_{t \rightarrow \infty} \int_1^t \frac{dx}{\sqrt{x}(1+x)}$ . Now  
 $\int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2u du}{u(1+u^2)}$  [ $u = \sqrt{x}$ ,  $x = u^2$ ,  $dx = 2u du$ ]  
 $= 2 \int \frac{du}{1+u^2} = 2 \tan^{-1} u + C = 2 \tan^{-1} \sqrt{x} + C$ ,  
so  $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \lim_{t \rightarrow 0^+} [2 \tan^{-1} \sqrt{x}]_t^1 + \lim_{t \rightarrow \infty} [2 \tan^{-1} \sqrt{x}]_1^t$   
 $= \lim_{t \rightarrow 0^+} [2(\frac{\pi}{4}) - 2 \tan^{-1} \sqrt{t}] + \lim_{t \rightarrow \infty} [2 \tan^{-1} \sqrt{t} - 2(\frac{\pi}{4})] = \frac{\pi}{2} - 0 + 2(\frac{\pi}{2}) - \frac{\pi}{2} = \pi$ .

59. First suppose  $p = -1$ . Then

$$\int_0^1 x^p \ln x dx = \int_0^1 \frac{\ln x}{x} dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{\ln x}{x} dx = \lim_{t \rightarrow 0^+} [\frac{1}{2}(\ln x)^2]_t^1 = -\frac{1}{2} \lim_{t \rightarrow 0^+} (\ln t)^2 = -\infty,$$

so the integral diverges. Now suppose  $p \neq -1$ . Then integration by parts gives

$$\int x^p \ln x dx = \frac{x^{p+1}}{p+1} \ln x - \int \frac{x^p}{p+1} dx = \frac{x^{p+1}}{p+1} \ln x - \frac{x^{p+1}}{(p+1)^2} + C. \text{ If } p < -1, \text{ then } p+1 < 0, \text{ so}$$

$$\int_0^1 x^p \ln x dx = \lim_{t \rightarrow 0^+} \left[ \frac{x^{p+1}}{p+1} \ln x - \frac{x^{p+1}}{(p+1)^2} \right]_t^1 = \frac{-1}{(p+1)^2} - \left( \frac{1}{p+1} \right) \lim_{t \rightarrow 0^+} \left[ t^{p+1} \left( \ln t - \frac{1}{p+1} \right) \right] = \infty.$$

61. (a)  $I = \int_{-\infty}^{\infty} x dx = \int_{-\infty}^0 x dx + \int_0^{\infty} x dx$ , and  
 $\int_0^{\infty} x dx = \lim_{t \rightarrow \infty} \int_0^t x dx = \lim_{t \rightarrow \infty} [\frac{1}{2}x^2]_0^t = \lim_{t \rightarrow \infty} [\frac{1}{2}t^2 - 0] = \infty$ , so  $I$  is divergent.  
(b)  $\int_{-t}^t x dx = [\frac{1}{2}x^2]_{-t}^t = \frac{1}{2}t^2 - \frac{1}{2}t^2 = 0$ , so  $\lim_{t \rightarrow \infty} \int_{-t}^t x dx = 0$ . Therefore,  $\int_{-\infty}^{\infty} x dx \neq \lim_{t \rightarrow \infty} \int_{-t}^t x dx$ .