

7.6 Integration Using Tables and Computer Algebra Systems

Keep in mind that there are several ways to approach many of these exercises, and different methods can lead to different forms of the answer.

1. We could make the substitution $u = \sqrt{2}x$ to obtain the radical $\sqrt{7 - u^2}$ and then use Formula 33 with $a = \sqrt{7}$.

Alternatively, we will factor $\sqrt{2}$ out of the radical and use $a = \sqrt{\frac{7}{2}}$.

$$\begin{aligned}\int \frac{\sqrt{7-2x^2}}{x^2} dx &= \sqrt{2} \int \frac{\sqrt{\frac{7}{2}-x^2}}{x^2} dx \stackrel{33}{=} \sqrt{2} \left[-\frac{1}{x} \sqrt{\frac{7}{2}-x^2} - \sin^{-1} \frac{x}{\sqrt{\frac{7}{2}}} \right] + C \\ &= -\frac{1}{x} \sqrt{7-2x^2} - \sqrt{2} \sin^{-1} \left(\sqrt{\frac{2}{7}} x \right) + C\end{aligned}$$

3. Let $u = \pi x \Rightarrow du = \pi dx$, so

$$\begin{aligned}\int \sec^3(\pi x) dx &= \frac{1}{\pi} \int \sec^3 u du \stackrel{71}{=} \frac{1}{\pi} \left(\frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| \right) + C \\ &= \frac{1}{2\pi} \sec \pi x \tan \pi x + \frac{1}{2\pi} \ln |\sec \pi x + \tan \pi x| + C\end{aligned}$$

12. Let $u = 3x$. Then $du = 3 dx$, so

$$\begin{aligned}\int x^2 \cos 3x dx &= \frac{1}{27} \int u^2 \cos u du \stackrel{85}{=} \frac{1}{27} (u^2 \sin u - 2 \int u \sin u du) \\ &\stackrel{82}{=} \frac{1}{3} x^2 \sin 3x - \frac{2}{27} (\sin 3x - 3x \cos 3x) + C \\ &= \frac{1}{27} [(9x^2 - 2) \sin 3x + 6x \cos 3x] + C\end{aligned}$$

Thus, $\int_0^\pi x^2 \cos 3x dx = \frac{1}{27} [(9x^2 - 2) \sin 3x + 6x \cos 3x]_0^\pi = \frac{1}{27} [(0 + 6\pi(-1)) - (0 + 0)] = -\frac{6\pi}{27} = -\frac{2\pi}{9}$.

24. Let $u = 2x$. Then $du = 2 dx$, so

$$\begin{aligned}\int \sin^6 2x dx &= \frac{1}{2} \int \sin^6 u du \stackrel{73}{=} \frac{1}{2} \left(-\frac{1}{6} \sin^5 u \cos u + \frac{5}{6} \int \sin^4 u du \right) \\ &\stackrel{73}{=} -\frac{1}{12} \sin^5 u \cos u + \frac{5}{12} \left(-\frac{1}{4} \sin^3 u \cos u + \frac{3}{4} \int \sin^2 u du \right) \\ &\stackrel{63}{=} -\frac{1}{12} \sin^5 u \cos u - \frac{5}{48} \sin^3 u \cos u + \frac{5}{16} \left(\frac{1}{2} u - \frac{1}{4} \sin 2u \right) + C \\ &= -\frac{1}{12} \sin^5 2x \cos 2x - \frac{5}{48} \sin^3 2x \cos 2x - \frac{5}{64} \sin 4x + \frac{5}{16} x + C\end{aligned}$$