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Math 1B Worksheet

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(Based on a worksheet by Rob Bayer)

Second Order Equations

1. For each of the following pairs of functions, determine whether they are linearly independent or dependent:

   (a) $f(x) = e^x,\ g(x) = e^{x+1}$

   (b) $f(x) = xe^{2x},\ g(x) = e^{2x}$

   (c) $f(x) = \ln(x^3),\ g(x) = \ln(x^\pi)$

   (d) $f(x) = \sin(x),\ g(x) = \cos(x)$

2. Which of the following second order differential equations are linear? Homogeneous?

   (a) $e^x y'' + \cos(3x^2)y' + 3y = 0$

   (b) $y'' + 3y' + 7y = \cos x$

   (c) $y'' + 3xy' + y^2 = 0$

   (d) $\tan(y'') + \cos(x)y' = e^x$

3. Consider the differential equation $y'' = -y$

   (a) By just thinking about it for a while, come up with two linearly independent solutions to this equation and then use them to find the general solution. (Think about elementary functions whose derivatives cycle.)

   (b) Show that $y = \cos(x + a)$ is a solution for any constant a.

   (c) Argue that parts (a) and (b) show that $\cos(x + a) = C_1 \cos x + C_2 \sin x$ for an appropriate choice of C_1 and C_2

   (d) Without using trig identities, find C_1 and C_2. (Hint: what should $y(0)$ and $y'(0)$ be? Remember that a is a constant, so C_1 and C_2 can refer to it.) Does this formula look familiar?

Solving Second Order Homogeneous Linear ODEs with Constant Coefficients

1. Find the general solution to $y'' + 3y' - 18y = 0$

2. Solve the initial value problem $y'' + 4y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 3$

3. Solve the boundary value problem $y'' = y$ with $y(0) = 0$ and $y(2) = 2$