(Problems selected from worksheets by Rob Bayer.)

(1) Determine whether each of the following sequences are convergent or divergent. For those that are convergent, find the limit.

(a) $a_n = \frac{3n^2+1}{n^2-1}$.

(b) $a_n = \frac{(n+2)!}{(2n)^2 \cdot n!}$.

(c) $\{1, \frac{1}{2}, 1, \frac{1}{4}, 1, \frac{1}{8}, \ldots\}$.

(d) $a_n = \ln(n^2 - 3n + 1) - \ln(n^2 + 4)$.

(e) $a_n = n \tan(1/n)$.

(2) True/False. For all problems, a_n and b_n are sequences. If the answer is true, cite a theorem, or explain why. If it is false, give a counterexample, i.e. two sequences for which it is false.

(a) If a_n and b_n converge, then $a_n + b_n$ converges.
(b) If $a_n + b_n$ converges, then a_n and b_n converge.
(c) If a_n and b_n converge, then a_n/b_n converges.
(d) If a_n and b_n diverge, then $a_n + b_n$ diverges.
(e) If $a_n + b_n$ diverges, then a_n and b_n diverge.
(f) If a_n and b_n diverge, then $a_n b_n$ diverges.

(3) For each of the following, give an example of a sequence with the required properties or explain why no such sequence can exist:

(a) Bounded, Monotonic, Convergent
(b) Bounded, Monotonic, Not Convergent
(c) Bounded, Not Monotonic, Convergent
(d) Bounded, Not Monotonic, Not Convergent
(e) Not Bounded, Monotonic, Convergent
(f) Not Bounded, Monotonic, Not Convergent
(g) Not Bounded, Not Monotonic, Convergent
(h) Not Bounded, Not Monotonic, Not Convergent

(4) More Sequences! Determine convergence or divergence, and calculate the limit if convergent:

(a) $a_n = \frac{\cos^2 n + n}{2^n + 3^n}$

(b) $a_n = \frac{n^{(-1)^n}}{n + \ln n}$

(c) $a_n = n^{\frac{\ln 2}{1 + \ln n}}$