Quiz 8

Math 1B, Spring 2012

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Section: Name:

Solve the differential equation:

$$\frac{dy}{dx} = y^2 \sin x.$$

For what values of the arbitrary constant will y be well-defined and continuous for all x?

We divide both sides by y^2, assuming $y \neq 0$. If $y = 0$, then $dy/dx = y^2 \sin x = 0$, so we include this as a possible solution. We integrate both sides with respect to dx, obtaining:

$$\int \frac{1}{y^2}\frac{dy}{dx}dx = \int \sin x dx.$$

$$\Rightarrow \int \frac{dy}{y^2} = -\cos x + C.$$

$$\Rightarrow -\frac{1}{y} = -\cos x + C.$$

$$\Rightarrow y = \frac{1}{\cos x + C}.$$

So the family of solutions for our differential equation is $y = 0$, and $y = \frac{1}{\cos x + C}$ for some constant C.

If we want the function to be well-defined everywhere, we need to make sure we never divide by zero. Since $\cos x$ takes all values between -1 and 1, we need $C < -1$ or $C > 1$.