Quiz 7

MATH 1B, SPRING 2012

March 16, 2012 SOLUTION GSI: Zvi Rosen

SECTION: NAME:

Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

$$f(x) = \frac{x+2}{2x^2 - x - 1}.$$

Factor: $2x^2 - x - 1 = (2x+1)(x-1)$

$$\frac{x+2}{2x^2-x-1} = \frac{A}{2x+1} + \frac{B}{x-1} \Rightarrow Ax - A + 2Bx + B = x + 2$$

$$\Rightarrow \begin{aligned} A + 2B &= 1 \\ -A + B &= 2 \end{aligned}$$

$$\Rightarrow B = 1, \; A = -1.$$

$$\Rightarrow \frac{x+2}{2x^2-x-1} = \frac{-1}{2x+1} + \frac{1}{x-1}$$

$$= -\frac{1}{1-(-2x)} + \frac{-1}{1-x}$$

$$= -\sum_{n=0}^{\infty} (-2x)^n + -\sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} (-1)^{n+1} 2^n x^n - \sum_{n=0}^{\infty} x^n$$

For the first series to converge, we need

$$|r| < 1 \Rightarrow |2x| < 1 \Rightarrow |x| < \frac{1}{2} \Rightarrow I = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

For the second to converge, we need $|r| < 1 \Rightarrow |x| < 1$. $I = (-1, 1)$

The sum converges on the overlap of the two series,

So $I = \left(-\frac{1}{2}, \frac{1}{2}\right)$