# Quiz 5

Math 1B, Spring 2012

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GSI: Zvi Rosen

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## Solution

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Determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}.$$

We use the limit comparison test:
Let $b_n = \frac{1}{n}$. Then $\frac{a_n}{b_n} = \frac{1}{n^{1+1/n}} \cdot n = \frac{1}{n^{1/n}}$.

To calculate the limit, we take the natural logarithm, recalling that, if $\{c_n\}$ converges to a limit where log is continuous, then:

$$\ln \lim c_n = \lim \ln c_n \Leftrightarrow \lim c_n = \exp(\lim \ln c_n).$$

So, for our sequence,

$$\lim_{n\to\infty} \ln \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{n} \ln\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{-\ln n}{n} = 0.$$

Therefore, by the logic above:

$$\lim \frac{a_n}{b_n} = \exp\left(\lim_{n\to\infty}\left(\ln\left(\frac{a_n}{b_n}\right)\right)\right) = 1.$$

By the limit comparison test, since the ratio converges to a finite limit greater than 0, either both series diverge or both converge.

Since the harmonic series diverges, we know that our original series diverges.