Quiz 2

MATH 1B, SPRING 2012

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SECTION: NAME: SOLUTION

Solve the following integral, showing all steps clearly:

$$\int_0^1 \frac{x\,dx}{x^2+4x+13}$$

$b^2 - 4ac = 16 - 4\cdot 13 < 0 \Rightarrow$ So, $x^2+4x+13$ is irreducible.

For a u-substitution $u = x^2+4x+13$

we need $du = (2x+4)\,dx$.

So, we take: $x = \frac{1}{2}(2x+4) - 2$.

$$\Rightarrow \int_0^1 \frac{x\,dx}{x^2+4x+13} = \underbrace{\int_0^1 \frac{\frac{1}{2}(2x+4)\,dx}{x^2+4x+13}}_{1} - \underbrace{\int_0^1 \frac{2\,dx}{x^2+4x+13}}_{2}.$$

For 1, we make the above u-sub: $= \frac{1}{2}\int_{13}^{18} \frac{du}{u}$.

$= \frac{1}{2}(\ln 18 - \ln 13)$.

For 2, we complete the square and do a trig sub:

$$\int_0^1 \frac{2\,dx}{x^2+4x+13} = 2\int_0^1 \frac{dx}{(x+2)^2+9} \begin{cases} = \frac{2}{3}\tan^{-1}\left(\frac{x+2}{3}\right)\Big|_0^1 \text{ (by the formula).} \\ = \frac{2}{3}\tan^{-1}(1) - \frac{2}{3}\tan^{-1}\left(\frac{2}{3}\right) = \frac{\pi}{6} - \frac{2}{3}\tan^{-1}\left(\frac{2}{3}\right) \end{cases}$$

Or by trig sub, $x+2 = 3\tan\theta$. when $x=1$, $\theta = \frac{\pi}{4}$

$dx = 3\sec^2\theta\,d\theta$ $x=0$ $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

$$= 2\int_{\tan^{-1}(\frac{2}{3})}^{\pi/4} \frac{3\sec^2\theta\,d\theta}{9\tan^2\theta+9} = 2\int_{\tan^{-1}(\frac{2}{3})}^{\pi/4} \frac{1}{3}\,d\theta = \frac{\pi}{6} - \frac{2}{3}\tan^{-1}\left(\frac{2}{3}\right).$$

So, $\displaystyle\int_0^1 \frac{x\,dx}{x^2+4x+13} = ① - ② = \frac{1}{2}(\ln 18 - \ln 13) + \frac{2}{3}\tan^{-1}\left(\frac{2}{3}\right) - \frac{\pi}{6}.$