Solve the following integral, showing all steps clearly:

$$
\int_{0}^{a} x^{2} \sqrt{a^{2}-x^{2}} \mathrm{~d} x
$$

