Midterm 2 Ma 53 Spr '08 Prof 

1. z is a function of x and y defined implicitly by $z^3 + z^2 x + z y^2 + x^2 y = 4$. Compute $z_x(1,1)$ and $z_y(1,1)$ assuming $z(1,1) = 1$.

$$3z^2 z_x + 2z z_x x + z^2 + z_x y^2 + 2xy = 0$$

@ $(1,1)$

$$3z_x + 2z_x + 1 + z_x + 2 = 0 \Rightarrow z_x = -\frac{1}{2}$$

$$3z^2 z_y + 2z z_y x + z_y y^3 + z\,2y + x^2 = 0$$

@ $(1,1)$

$$3z_y + 2z_y + z_y + 2 + 1 = 0 \quad z_y = -\frac{1}{2}$$

2. $x(t)$ satisfies $x'(t) = u(x(t), t)$,

where $u(x,t)$ is a given function

with $u(1,1) = 1$, $u_t(1,1) = 1$, $u_x(1,1) = 1$,

and $x(1) = 1$. Compute $x''(1)$.

$$x''(t) = u_t(x(t), t) + u_x(x(t), t)\, x'(t)$$

$$= (u_t + u\, u_x)(x(t), t)$$

@ $t = 1$, $x''(t) = 1 + 1 \cdot 1 = 2$.

3. a) Compute $\underline{\nabla} f$ for $f(x, y, z) =$

$z^3 + z^2 x + z y^2 + x^2 y$ at $(x, y, z) =$

$(1, 1, 1)$.

$$\underline{\nabla} f = (z^2 + 2xy)\,\underline{i} + (2zy + x^2)\,\underline{j} + (3z^2 + 2zx + y^2)\,\underline{k} =$$

@ $(1,1,1)$

$$3\underline{i} + 3\underline{j} + 6\underline{k}$$

b) What is the equation of the tangent plane to the surface $f(x, y, z) = 4$ at $(x, y, z) = (1, 1, 1)$?

$$(3\underline{i} + 3\underline{j} + 6\underline{k}) \cdot ((x-1)\underline{i} + (y-1)\underline{j} + (z-1)\underline{k}) = 0$$

$$3(x-1) + 3(y-1) + 6(z-1) = 0$$

c) Compute the linearized approx-

imation to $f(1.01, 1.02, 1.005)$.

$f(1.01, 1.02, 1.005) \cong$

$f(1,1,1) + f_x(1,1,1)(.01) +$

$f_y(\quad)(.02) +$

$f_z(\quad)(.005) =$

$4 + 3(.01) + 3(.02) + 6(.005) = 4.12.$

$.03 \qquad .06 \qquad .03$

4. Compute $\iint\limits_D e^{-x} e^{-y} \, dA$,

where D is the region bounded by the line $x + y = s$ = positive constant, and the x and y axes.

$$\int_0^s e^{-x} \left(\int_0^{s-x} e^{-y} \, dy \right) dx =$$

$$\int_0^s e^{-x} \left(1 - e^{x-s} \right) dx =$$

$$\int_0^s \left(e^{-x} - e^{-s} \right) dx = 1 - e^{-s} - s e^{-s}.$$

5. Compute $\displaystyle\iiint_E \frac{dV}{\sqrt{x^2+y^2+(r-z)^2}}$

where E is the "ball" $0 < \rho < R$ and

r is a constant, $r > R$.

$$2\pi \int_0^R \int_0^\pi \frac{\rho^2 \sin\varphi}{\underbrace{\sqrt{\rho^2 \sin^2\varphi + (r - \rho\cos\varphi)^2}}_{r^2+\rho^2-2r\rho\cos\varphi}}\, d\varphi\, d\rho =$$

$$2\pi \int_0^R \frac{\rho^2}{2r\rho}\left[\sqrt{r^2+\rho^2-2r\rho\cos\varphi}\,\right]_0^\pi d\rho =$$

$$\frac{\pi}{r}\int_0^R \rho\{ r+\rho - r+\rho \} d\rho =$$

$$\frac{2\pi}{r}\int_0^R \rho^2\, d\rho = \frac{2\pi}{3r} R^3$$