

Harder Problems

1. Given a line and a point, find the point on the line that is closest to that point.
2. 12.5.73. Hint: This is an application of formula 9 in that section.
3. 12.5.75. Hint: you can either use 12.5.73 or vector projection.
4. 12.3.49. Hint: is the vector $\langle a, b \rangle$ parallel or perpendicular to the line? Why?
5. Let $P = \langle x, y, z \rangle$. Let α, β, γ be the angles between OP and the xy, yz, zx -planes (OP means the vector from the origin to P). Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 2$.
Hint: how do you calculate the angle between a plane and a vector? What's the projection of OP onto, say, the xy -plane?
6. Given a regular tetrahedron with three of the four vertices being $(0, 0, 0), (2, 0, 0), (1, 1, \sqrt{2})$. What's the coordinate of the last vertex?
Hint: What's the defining property of a "regular" tetrahedron?
7. 10.2.74. Hint: first figure out a way to parametrize the boundary of the region that the cow can reach. The angle formed by the tangent point and the x-axis isn't a bad choice.