# Errata for Understanding Numbers in Elementary School Mathematics

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**Page xxxi:** Insert the following at the end of the second bullet.

However, note that not all **boldfaced** words or phrases in the text are definitions because sometimes we use boldfaced fonts for emphasis.

**Page xxxi:** line −11.
"Page 236" should be "Page 210".

**Page 10:** lines 1 & 2.
"hundreds place" should be "leftmost place" (*twice*).

**Page 16:** replace lines −5 & −4 (not counting the footnote) with:

900,000) takes the same number of steps as the passage from 0 to 100,000, which is exactly 100,000 steps.

**Page 18:** line 8. "$173 \to 174 \to 175 \to$" should be
"$172 \to 173 \to 174 \to$"

**Page 19:** $(i)$ line $-14$. "3 2" should be "32".

$(ii)$ Line $-12$. "then 8 in 28" should be "the 8 in 28".

**Page 19:** lines $-6$ & $-5$.
"To count to the eighth row, one must first count to 700," should be
"To count to 728, one must first count to 700 in the eighth row,"

**Page 22:** line 3. "pages 4–6]" should be "pages 4–6])"

**Page 28:** $(i)$ Line 15.
"3 added to itself five times" should be "the sum of five copies of 3"

$(ii)$ line 16.
"5 added to itself three times" should be "the sum of three copies of 5"

**Page 31:** line 17.

10 times the *place value* of the digit to its right

should be

10 times the *place value* of this same digit if it were put in
the next place to its right

**Page 32:** $(i)$ Middle of page.
"$\boldsymbol{n \times 10^{k}}$" should be "$n \times 10^{k}$". (no boldface)

$(ii)$ Lines $-6$ & $-5$. Replace

$$0 \times 10^8 + 0 \times 10^7 + 0 \times 10^6 + 8 \times 10^5$$
$$+ 3 \times 10^4 + 1 \times 10^2 + 5 \times 10^1 + 9 \times 10^0$$

by

$$(0 \times 10^8) \ + \ (0 \times 10^7) \ + \ (0 \times 10^6) \ + \ (8 \times 10^5) \ + \ (3 \times 10^4)$$
$$+ \ (0 \times 10^3) \ + \ (1 \times 10^2) \ + \ (5 \times 10^1) \ + \ (9 \times 10^0)$$

**Page 35:** Exercise 7. First line: "page 26" should be "pp. 26-27".

**Page 43:** $(i)$ Line 4. Delete "(Theorem 2.1)".

$(ii)$ Replace the two activities near the middle of the page by:

**Activity.** Compute $37 + 189 + 163$ and
$275 + 892 + 225 + 4211 + 108$.

**Activity.** Compute $666{,}666{,}667 + 788{,}646{,}851{,}086 + 333{,}333{,}333$.

**Page 44:** $(i)$ Line 12. "876190" should be "871690".

$(ii)$ Line $-6$.
"$\boldsymbol{n \times (m \times 10^k)}$" should be "$n \times (m \times 10^k)$". (no boldface)

**Page 45:** the displayed equations on lines 8 and 10 should not be in boldface.

**Page 46:** This page has several corrections.

$(i)$ The displayed equation in the middle of the page should not be in boldface.

$(ii)$ Replace the first four lines of the **Activity** *by:*

One can use the distributive law to multiply a one-digit number by a two-digit number using mental math. For example, to compute $6 \times 43$, we break up $43$ into $(40 + 3)$ so that

$6 \times 43 = 6 \times (40 + 3) = (6 \times 40) + (6 \times 3)$, and the last sum is just $240 + 18 = 258$.

(*iii*) **Replace** the last two lines of the page *by:*

Therefore, $6 \times 43 = 258$. Now follow this example and use mental math to compute: (a) $8 \times 24$, (b) $7 \times 53$, (c) $6 \times 39$, (d) $5 \times 79$, (e) $9 \times 94$, (f) $8 \times 47$.

**Page 47:** line 4. "$\boldsymbol{N \times 10^k}$" should be "$N \times 10^k$". (no boldface)

**Page 58:** line 3. "this is 12 added to itself 17 times" should be "this is the sum of 17 copies of 12".

**Page 62:** line $-2$ (not counting the footnotes). "resulting numbers" should be "resulting number".

**Page 64:** (*i*) Lines $-11$ & $-10$. "the sum of the digits in the second column" should be "10 times the sum of the digits in the second column".

(*ii*) Lines $-4$ & $-3$. "the sum of the digits in the third column" should be "100 times the sum of the digits in the third column".

**Page 67:** line 9. "exceeds 10" should be "exceeds 9".

**Page 70:** line 3. "nine even numbers are 2, 4, 6" should be "ten even numbers are 0, 2, 4, 6".

**Page 72:** (*i*) The displayed equality on line $-12$, namely,

$$k + n \;=\; m$$

should be

$$n + k \;=\; m$$

($ii$) Lines $-2$ & $-1$. Delete all the "$= ?$".

**Page 79:** below the 9th line, i.e., below the line

$$11 - 9 \quad 11 - 8 \quad 11 - 7 \quad 11 - 6 \quad 11 - 5 \quad 11 - 4 \quad 11 - 3 \quad 11 - 2$$

insert:

$$10 - 9 \quad 10 - 8 \quad 10 - 7 \quad 10 - 6 \quad 10 - 5 \quad 10 - 4 \quad 10 - 3 \quad 10 - 2 \quad 10 - 1$$

**Page 83:** there are several corrections.
($i$) Exercise 4, second line.
"he and Ben" should be "himself and Ben".

($ii$) Exercise 10. Delete all the "$= ?$".

($iii$) Exercises 10 should be placed after Exercise 12.

**Page 84:** Exercise 13, second line, "Exercise 12(i)" should be "Exercise 12(iii)".

**Page 86:** the third line of footnote 1.
"memorization the $12 \times 12$" should be "memorization of the $12 \times 12$".

**Page 92:** line $-4$. Replace "15" by "17".

**Page 93:** delete Exercise 9.

**Page 103:** line $-12$. Replace "$q$ as that particular multiple $qd$ that equals $a$" by

> $q$ as the whole number so that the $q$-th multiple of $d$—which is $qd$—is equal to $a$

**Page 104:** line $-7$. "first $q$ multiples" should be
"first $q$ positive multiples".

**Page 105:** line 4.
"whole numbers $a$ and $d$ with $d > 0$" should be
"whole numbers $a$ and $d$ with $d > 0$ and $a$ not a multiple of $d$".

**Page 107:** lines $-13$ & $-12$. Replace "so that the consecutive multiples . . . 586 between them." by

> so that 586 is either equal to $q \times 3$ or trapped between $q \times 3$ and $(q + 1) \times 3$.

**Page 108:** line $-14$. "*divisions-with-remainders*" should be "*divisions-with-remainder*".

**Page 110:** line $-1$ and also line $-16$. "divisions-with-remainders" should be "divisions-with-remainder".

**Page 112:** line $-10$ & $-9$ (not counting the footnote). Replace "Recall that $q$ is the largest multiple of 3 . . . equal to 586" by

> Recall that q is the largest whole number so that $q$ times 3 is less than or equal to 586

**Page 115:** line $-2$. "Finally, give" should be ""Finally, we give".

**Page 116:** lines 6&7 (ignoring the "division houses"). Replace "Recall that $q$ is the largest multiple . . . equal to 1308." by

> Recall that $q$ is the largest whole number so that $q$ times 35 is less than or equal to 1308.

**Page 117:** (*i*) Line 9. "dividend 195" should be "quotient 195".

(*ii*) Line −8. "divisions-with-remainders" should be "divisions-with-remainder".

**Page 118:** line −15. "divisions-with-remainders" should be "divisions-with-remainder".

**Page 119:** (*i*) Middle of page, third line of section 7.6. "divisions-with-remainders" should be "divisions-with-remainder".

(*ii*) line −7.
"the largest multiple of the divisor *not to exceed*" should be
"the largest whole number $q$ so that $q$ times the divisor $\leq$ ".

**Page 120:** this page has several corrections.
(*i*) Line 8. "the dividend of 129" should be "the quotient of 129".

(*ii*) Line 17. "*largest multiple of* 35 *not to exceed*" should be
"*largest whole number $q$ so that* $q \cdot 35 \leq$ ".

(*iii*) Line −8 &−9. "*largest multiple of d not to exceed*" should be
"*largest whole number $q$ so that $qd \leq$* ".

**Page 123:** (*i*) Exercise 3, lines 3 and 4. Replace

"1234,5497,2086" by "123,454,972,086" (*twice*), and

"8026,5937" by "80,265,937".

(*ii*) Delete Exercise 4.

**Page 124:** (*i*) Exercise 14. Replace
"division of 652 by 8"  by  "division-with-remainder of 652 by 8".

(*ii*) Second line of Exercise 16.
"divisions-with-remainders"  should be  "divisions-with-remainder".

**Page 128:** There are several corrections on this page.
(*i*) Line 11. Replace   "to the right of 0"  by  "that begins with 0 and goes to the right of 0".

(*ii*) The first line of the **Definition**  should be
"*A **whole number** is the point* 0 *or one of the marked points* ..."

(*iii*) Line 5 of the **Definition**. "the whole numbers on it"  should be  "the whole numbers located on it as described".

(*iv*) lines −9 to −8.  "We also identify a whole number $n$ ... from 0 to $n$."  should be  "Henceforth, a whole number $n$ will stand for *both* the point on the number line and the length of the segment $[0, n]$ from 0 to $n$."

**Page 129:** line 3.
"interpreted in terms of"  should be "defined in terms of".

**Page 133:** line 3. "$0 \div 0 =$?"  should be "what is $0 \div 0$ ?".

**Page 144:** (*i*) Line 11. "$127 + 284 =$?"  should be "$127 + 284$.".

(*ii*) Line −2.  "and but"  should be "but".

**Page 145:** lines −16 to −15.
"by any definition of"  should be "by any everyday definition of".

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**Page 146:** line 7. "**5%**" should be "5%". (no boldface)

**Page 147:** there are several corrections on this page.
($i$) Line 1. "$7,500,000,000" should be "$7,500,000,000,000".

($ii$) line 13. "is 102,743" should be

is listed as 102,743 (as of year 2010)

($iii$) line $-19$. "*at the least*" should be "*at the most*".

**Page 151:** line 4. "accuracy up to 1.2 meters" should be "with an error of no more than 1.2 meters".

**Page 156:** ($i$) Line 12. Replace

whole number $b$, *using only multiples $< b$.*

by

whole number $b > 1$, with the restriction that in any such multiple $q\, b^m$, where $q$ and $m$ are whole numbers, *we require $q < b$.*

($ii$) First line of the last paragraph. "$b$ is a positive whole number" should be "$b$ is a whole number $\geq 2$".

**Page 157:** ($i$) Lines $-12$ & $-11$.
"in computer science" should be "in computers".

($ii$) Line 9. "6 exceeds 5" should be "$6 \geq 5$.

**Page 158:** line $-9$. "$17 > 7$ should be "$17 \geq 7$".

**Page 159:** This page needs many corrections.

($i$) Line 6. "$\underline{1}$" should be "1".

($ii$) Between lines 6 and 7, insert the following displayed equation:

$$1 = 0 \cdot 7 + 1$$

($iii$) Line 7. Replace "Because $1 < 7$ in the last equation, the process stops. Also observe" by

> The last equation shows that there is no need to continue. Observe

($iv$) Line 8. "divisor (i.e., 4, 2, 4, 3)" should be "divisor (i.e., 4, 2, 4, 3, 1)".

($v$) Line 10. "we get" should be "we have gotten".

($vi$) Lines $-15$ & $-14$. Replace "Notice that among the coefficients ... of 3644, {3, 4, 2, 4} are" by

> Notice that the coefficients 1, 3, 4, 2, 4 of the base 7 representation of 3644 are

($vii$) Line $-14$ & line $-13$. "divisions-with-remainders" should be "divisions-with-remainder".

($viii$) Line $-7$. Replace "We see that 3644" by:

> If the number 3644 were equal to $7^n$ for some whole number $n$, for example, $7^4$, then $3644 = 1 \cdot 7^4 + 0 \cdot 7^3 + 0 \cdot 7^2 + 0 \cdot 7^1 + 0$ and we would have the base 7 representation of 3644 as $(10000)_7$. But, of course, 3644 actually

($ix$) Line $-3$. "falls between $7^3$ and $7^4$" should be "$< 7^4$".

**Page 160:** lines $-9$ & $-8$.
"divisions-with-remainders" should be "divisions-with-remainder".

**Page 161:** lines 1 & 2.
"divisions-with-remainders" should be "divisions-with-remainder".

**Page 162:** change the **Activity** to:

Compute: $(66)_7 + (1)_7, \ (2666)_7 + (1)_7, \ (266660)_7 + (10)_7$.

**Page 162:** the subtraction near the bottom of the page,

$$
\begin{array}{r}
\phantom{-} \ \ ^4\phantom{5} \ \ ^6\phantom{0} \ \ \phantom{2} \\
\not{5} \ \ \not{0} \ \ 2 \\
- \ \ \ \ 2 \ \ 1 \ \ 3 \\
\hline
2 \ \ 5 \ \ 6
\end{array}
$$

should be (the changes are indicated in red):

$$
\begin{array}{r}
\phantom{-} \ \ ^4\phantom{5} \ \ ^6\phantom{0} \ \ ^{12}\phantom{2} \\
\not{5} \ \ \not{0} \ \ \not{2} \\
- \ \ \ \ 2 \ \ 1 \ \ 3 \\
\hline
2 \ \ 5 \ \ 6
\end{array}
$$

**Page 163:** there are several corrections.
($i$) The first line below the first **Activity**:
"no difference" should be "no formal difference".

($ii$) Replace the second **Activity** by:

Compute: $(345)_7 \times (10)_7, \ (345)_7 \times (100)_7, \ (345)_7 \times (10000)_7$.

(*iii*) **Replace** the first line below the second **Activity** by:

We now come to the feature that is common to the multiplication algorithm in both base 10 and base

**Page 164: Replace** the first sentence of section 11.4 by:

It was remarked earlier that computers use the binary numeral system.

**Page 165: Replace** lines $-3$ & $-2$. by:

for any whole numbers $m$ and $n$, because the left side is equal to

$$\underbrace{2^m + \cdots + 2^m}_{2^n} \;=\; 2^n \times 2^m \;=\; 2^{m+n},$$

which is exactly the right side. For $m = 0$ and $n = 2$, we have the following special case:

**Page 166:** (*i*) **Replace** the displayed equation on line 14, i.e.,

$$(\underbrace{11\cdots1}_{m})_2 + (1)_2 = (1\underbrace{00\cdots0}_{m})$$

by the following (the change is indicated in red):

$$(\underbrace{11\cdots1}_{m})_2 + (1)_2 = (1\underbrace{00\cdots0}_{m})_2$$

(*ii*) The computation on line 18,

$$
\begin{array}{cccccc}
 & & 1 & 1 & 1 & 1 \\
 & & & & & 1 \\
+ & & {}_1 & {}_1 & {}_1 & \\
\hline
 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

should be  (the change is indicated in red):

$$
\begin{array}{cccccc}
 & & 1 & 1 & 1 & 1 \\
 & & & & & 1 \\
+ & {}_1 & {}_1 & {}_1 & {}_1 & \\
\hline
 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

**Page 167:** the last computation on the page, i.e.,

$$
\begin{array}{ccccccc}
 & & & 1 & 1 & 1 & 1 \\
\times & & & & & 1 & 1 \\
\hline
 & & & 1 & 1 & 1 & 1 \\
 & & 1 & 1 & 1 & 1 & \\
+ & & {}_1 & {}_1 & {}_1 & & \\
\hline
 & 1 & 0 & 1 & 1 & 0 & 1
\end{array}
$$

should be (the change is indicated in red):

$$
\begin{array}{ccccccc}
 & & & 1 & 1 & 1 & 1 \\
\times & & & & & 1 & 1 \\
\hline
 & & & 1 & 1 & 1 & 1 \\
 & & 1 & 1 & 1 & 1 & \\
+ & {}_1 & {}_1 & {}_1 & {}_1 & & \\
\hline
 & 1 & 0 & 1 & 1 & 0 & 1
\end{array}
$$

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**Page 168:** there are several corrections on this page.

($i$) Exercises 3 and 4.
"scientific calculator" should be "4-function calculator".

($ii$) Exercise 6 (a) and (c).
"$1 + 2^2 + 2^3 + 2^4 + 2^5 = 2^6 - 1$" should be
"$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 = 2^6 - 1$". (*twice*)

($iii$) Replace Exercise 7 by:

Compute: $(111)_2 + (111)_2 + (111)_2 + (111)_2$.

($iv$) Exercise 8(d). Insert at the end, "(Here, $(b-1)(b-1)\cdots(b-1)$ is not the product of the $(b-1)$'s but is, rather, a collection of digits of a certain number in base $b$.)".

**Page 181:** line $-7$.
"page 86 of Chapter 7" should be "page 99 of Chapter 7".

**Page 182:** ($i$) Line 8. Delete "completely".

($ii$) Line 9. "the right endpoint" should be "its right endpoint."

**Page 183:** line 14. "this chapter" should be "Part 2".

**Page 184:** ($i$) Line $-4$. "*nonzero* whole number" should be "whole number".

($ii$) Line $-3$. "1, 2, 3, 4, ..." should be "0, 1, 2, 3, 4, ...".

**Page 185:** ($i$) Line 6.

"for any whole number $m > 0$ and $\ell > 0$"  should be
"for any whole numbers $m$ and $\ell$, with $\ell > 0$".

($ii$) Line $-3$. Replace  "Also, we adopt" by:
"We also say  $\frac{m}{n}$  is $\boldsymbol{m}$ $\textbf{over}$ $\boldsymbol{n,}$ and we adopt".

**Page 186:** ($i$) Line 6. "is a whole number"  should be
"was a whole number".

($ii$) Replace  line 15 ("This is the terminology that will be used often in the rest of the chapter.") by the following:

> To the extent that we identify a fraction $\frac{m}{n}$ with the segment $[0, \frac{m}{n}]$, we will abuse the language and use the same sentence to express
>
> > $[0, \frac{m}{n}]$ is the concatenation of $m$ segments, each of length $\frac{1}{n}$.
>
> This is the terminology that will be used in the rest of Part 2.

**Page 188:** line 11.  "We will do that in the next section" should be  "We will do that in section 13.2".

**Page 200:** line 7.
"48"  should be  "47" (*twice*), and "203"  should be  "202".

**Page 205:** line $-7$.  " $\frac{38}{57} = \frac{3}{2}$"  should be  " $\frac{38}{57} = \frac{2}{3}$".

**Page 210:** lines 8–11.
Replace  "In the interest of brevity, ... Now divide" by

"Using the terminology of *equal parts* defined on page 180, we divide".

**Page 211:** line 8 (the line below the second picture). "The unit interval $[0, 1]$ is the concatenation of three copies of length $\frac{1}{3}$," should be "The unit interval $[0, 1]$ is the concatenation of three segments of length $\frac{1}{3}$".

**Page 211:** line $-14$. "$\frac{5}{3}$ is $5 \times 4$" should be "$\frac{4}{3}$ is $4 \times 5$".

**Page 213:** replace the five indented lines in the middle of the page, below "**Pairs (FFFP)**", by:

> *Any two fractions can be denoted by fraction symbols which have the same denominator.*

For example, if the given fractions are $\frac{k}{\ell}$ and $\frac{m}{n}$, where $k$, $\ell$, $m$, $n$ are whole numbers $(\ell, n \neq 0)$, then they are respectively also equal to

$$\frac{kn}{\ell n} \quad \text{and} \quad \frac{\ell m}{\ell n}$$

**Page 214:** lines 9–18. Delete:
"We hasten to add that ... the same decimals as before."

**Page 219:** line 4 of Exercise 11.
"At no time" should be "At no other time".

**Page 221:** line $-11$. Replace this line by:

the addition of fractions in general. Now, for any fraction $A$, $\mathbf{0} + \mathbf{A}$ and $\mathbf{A} + \mathbf{0}$ are defined to be $A$. Therefore to simplify matters, *the following discussion of the addition of two fractions will tacitly assume that both fractions are nonzero.*

**Page 222:** equation (14.3) should not be in boldface.

**Page 224:** lines 5 & 6.

" $\frac{k}{l}$ " should be " $\frac{k}{\ell}$ ".

" $\frac{k+m}{l+n}$ " should be " $\frac{k+m}{\ell+n}$ ".

" $l = n = 2$ " should be " $\ell = n = 2$ ".

**Page 225:** $(i)$ Line 5. Replace

**complete expanded form** of 4.1297.

by

*complete expanded form* of 4.1297.

$(ii)$ Line $-15$ *and* line $-11$. " $\frac{r}{l}$ " should be " $\frac{r}{\ell}$ ".

**Page 230:** Replace the first sentence of section 14.6 by:

If $n$ is a nonzero whole number, we define $\boldsymbol{n!}$ (read: $\boldsymbol{n}$ **factorial**) to be the product of all the whole numbers from 1 to $n$. We also define $\boldsymbol{0!}$ to be 1.

**Page 231:** line 1. " $\binom{n}{k}$ " should be " $\binom{\boldsymbol{n}}{\boldsymbol{k}}$ ". (boldface)

**Page 232:** there are several corrections.
$(i)$ Exercise 3. Delete " $= ?$ " in (a). *Replace* (b) by:
"Compute: $0.57 + 14.3 + 27.0802$."

$(ii)$ Exercise 4. Insert "Compute:" at the beginning, and *delete* all the " $= ?$ ".

$(iii)$ Exercise 5. Delete both " $= ?$ ".

(*iv*) Exercise 11. The second line: "$[4\frac{1}{3}, E]$" should be "$[3\frac{1}{3}, E]$".
Also *replace* the $4\frac{1}{3}$ in the number-line picture by $3\frac{1}{3}$.

**Page 236:** replace the first line below equation (15.1) by:

(Recall the definition on page 180: *equal parts* in the context
of the"

**Page 237:** lines 15-17 (the three lines immediately below the displayed equation):

so that $[0, m]$ is exhibited as $nm$ copies of $\frac{1}{n}$. Thus the
segments, so that each consists of $m$ copies of $\frac{1}{n}$, divide $[0, m]$
into $n$ equal parts; it follows that each of these $n$ parts is $m$
copies of $\frac{1}{n}$. i.e., $\frac{m}{n}$ as desired.

should be

so that $m$ is exhibited as $nm$ copies of $\frac{1}{n}$. Thus, $[0, m]$ is
divided into $n$ equal parts, each part consisting of $m$ copies
of $\frac{1}{n}$, which is $\frac{m}{n}$.

**Page 244:** line 16. "by Example 4" should be
"by Theorem 15.1 on page 240".

**Page 246:** line 15 (near the middle of the page). "what was proved
in (15.1)" should be "what was proved in Section 15.1".

**Page 246:** The Activity at the bottom of the page. Add:

Your explanation for (b) should be directly in terms of the
*mixed number* $3\frac{3}{7}$.

**Page 247:** lines 9-10. "where each segment is itself the concatenation of" should be "each of which is".

**Page 247:** line15. "take one part." should be "take the length of one part."

**Page 254:** line 7. " $\frac{k}{l}$ " should be " $\frac{k}{\ell}$ " and " $\frac{kn}{ln}$ " should be " $\frac{kn}{\ell n}$ ".

**Page 258:** There are several corrections on this page.
($i$) The third line of Exercise 4. " $\frac{7}{18}$ " should be " $\frac{17}{18}$ ".

($ii$) Exercise 5. Insert "Compute:" at the beginning and delete all the "= ?".

($iii$) The second line of Exercise 7. "and 1" should be "or 1".

**Page 264:** bottom line. " $\frac{1}{\ell}$ " should be " $\frac{1}{7}$ ".

**Page 269:** line −16. "the definition of a decimal" should be "the definition of a (finite) decimal".

**Page 274:** line −13. "when $\frac{k}{\ell}$ is divided into $n$ equal parts." should be "when $[0, \frac{k}{\ell}]$ is divided into $n$ equal parts."

**Page 275:** line 2. "when $\frac{k}{\ell}$ is divided into $n$ equal parts." should be "when $[0, \frac{k}{\ell}]$ is divided into $n$ equal parts."

**Page 275:** line −8. "plus the totality of $b$ parts when $\frac{k}{\ell}$ is divided into $c$ equal parts." should be "plus the total length of $b$ parts when $[0, \frac{k}{\ell}]$ is divided into $c$ equal parts."

**Page 277:** line 9. " $\frac{39}{15}$ " should be " $\frac{38}{15}$ ".

**Pages 280:** This page has several corrections.
($i$) Exercise 2. Delete all the "= ?".

($ii$) Exercise 3. Insert "Compute:" at the beginning, and *delete* all the "= ?".

($iii$) Exercise 4. Replace (a) by: "Compute: $\frac{3}{10} + \frac{5}{12}$ ."

($iv$) Exercise 8. Replace first sentence by: "Compute $8\frac{2}{50} \times 1250\frac{1}{2}$."

**Page 281:** ($i$) Exercise 13. Insert "Compute:" at the beginning and delete all the "= ?".

($ii$) Exercise 17. Last line: "Consult section 15.1" should be "Consult section 15.2".

**Page 284:** This page has several corrections.
($i$) Line 1. "We begin with" should be "We begin by recalling".

($ii$) In Definiton 2, $\frac{m}{n}$ should be $\boldsymbol{m \div n}$.

($iii$) Line $-4$. "$m > n$" should be "$m \geq n$", and "$A > B$" should be "$A \geq B$"

**Page 287:** line $-7$ (not counting the number-line picture).

"$\frac{7}{3} = \frac{5}{4} \times C$" should be "$\frac{7}{3} = C \times \frac{5}{4}$"

**Page 289:** delete the last paragraph (line $-11$ to line $-1$), because it is superfluous.

**Page 290:** line 1. Replace "As before, we note that," by "We note, as in the case of whole numbers (see page 286), that".

**Page 291:** lines $-6$ and $-5$. Replace

...nonzero whole number $n$,

$$\frac{1}{\frac{1}{n}} \ = \ n.$$

by

...nonzero whole number $\ell$,

$$\frac{1}{\frac{1}{\ell}} \ = \ \ell.$$

**Page 293:** line $-10$. "for any $t$" should be "for any $t > 0$".

**Page 294:** there are several corrections.
$(i)$ Line 1. "being 66 mph" should be "is 66 mph".

$(ii)$ Line 2. "in any time interval" should be "in all time intervals".

$(iii)$ Line 6. "it will *all* be 48 mph." should be
"will *always* be 48 mph."

$(iv)$ Line 11. "$s$ miles." should be "$s$ feet."

**Page 295:** line 2. "then equality" should be "the equality".

**Page 296:** $(i)$ Line 3. "$m$" should be "$T$".

(*ii*) Lines −13 & −12.  replace  "by Chapter 15 ... at the end of section 18.2" by "the discussion at the end of section 15.2".

**Page 297:** lines −4 & −3 (not counting footnotes).  "illustrated in ... page 177"  should be  "in equation (17.5) on page 271".

**Page 299:** line −8.  "Chapter 17"  should be
"equation (17.5) on page 271".

**Page 300:** line 15.  "is worthwhile"  should be "it is worthwhile".

**Page 302:** (*i*) Line 16.  "*the fraction* $\frac{2}{7}$"  should be  "*the fraction*".

(*ii*) Line −4.  "the only hand"  should be  "the other hand".

**Page 303:** line −9 (not counting footnote).  "associates"  should be
"associates with".

**Page 306:** (*i*) Exercise 5(b).  "If $\frac{4}{7}$ of a sack of rice"  should be
"If $\frac{4}{7}$ of a sack of rice (by weight)".

(*ii*) Exercise 9.  Add the assumption that *Shawna drives at a constant speed.*

**Page 307:** there are several corrections.
(*i*) Line 1.  "same speed"  should be  "same constant speed".

(*ii*) Exercise 10.  "60 mph"  should be  "at a constant speed of 60 mph".  "70 mph"  should be  "at a constant speed of 70 mph".

(*iii*) Exercise 14, second line.  "for a definition"  should be
"for the definition".

($iv$) Exercise 16 (a). *Insert* "Compute:" at the beginning, and delete both "= ?".

**Page 310:** line 7. Delete "*virtually*".

**Page 311:** line 7. "(a) $\leftrightarrow$ (17.4)" should be "(a) $\leftrightarrow$ equation above (17.3)".

**Page 314:** ($i$) Line 1. Replace "of course, but" by "of course, by letting $A = \frac{k}{\ell}$, $B = \frac{m}{n}$, and using invert-and-multiply to compute, but".

($ii$) Line $-3$ (not counting the number-line picture). Delete "($b$)".

**Page 315:** ($i$) Line $-12$ (not counting the footnote). Delete "= ?".

($ii$) Lines $-10$ & $-9$. Replace "in the sense that" by "because".

**Page 316:** in the second displayed equation near the middle of the page, the left side

$$\frac{1}{1 - \frac{2}{3}} + \frac{1}{1 - \frac{2}{3}}$$

should be

$$\frac{1}{1 - \frac{2}{3}} + \frac{1}{1 + \frac{2}{3}}$$

**Page 317:** second line of Exercise 4. "(15.2) on page 240" should be "(14.5) on page 228".

**Page 320:** line $-13$. "to to" should be "to".

**Page 323:** line $-12$. "$153\frac{1}{3}$" should be "$155\frac{5}{9}$".

**Page 324:** line $-4$. "If the low price" should be "If the high price".

**Page 325:** line 2. Delete the lower brace at the end of the line.

**Page 326:** There are several corrections.
($i$) Line 11. "**absolute error**" should be "absolute error (see page 144 for the definition)".

($ii$) Lines 13, and 15. Replace "*true value*" by "*correct value*".

($iii$) Line 14. "true value" should be "correct value".

**Page 328:** ($i$) Exercise 3.
"20%" on the fourth line of the exercise should be "15%"

($ii$) Exercise 11. Second line, "went the game" should be "went to the game".

**Page 332:** ($i$) Add the following to the indented paragraph in the middle of the page on **FASM**: "(A more precise statement of **FASM** is given on page 200 of the author's *Teaching School Mathematics: Pre-Algebra*, Amer. Math. Soc., Providence, RI, 2016. `http://tinyurl.com/y6ulr5kq`.)"

($ii$) Second line below the indented paragraph in the middle of the page. Replace "as if they were integers" by "as if they were fractions".

**Page 333:** line 6. "the real number" should be "the unique real number".

**Page 337:** replace lines $-8$ to $-4$ (not counting the footnote), inclusive, with the following:

> We now point out that this $U$ must in fact be a *whole number*, and the reason depends on Theorem 35.2 on page 470 (because the proof of this theorem does not involve any facts about fractions, there is no circular reasoning). Since $B/4 = G/5$, the cross-multiplication algorithm implies that $5B = 4G$. Thus 4 divides the right side ($4G$) and therefore also the left side ($5B$). But 4 and 5 are relatively prime, so 4 dividing $5B$ implies that 4 divides $B$, by Theorem 35.2. Similarly, 5 divides $G$. We may therefore conclude:

**Page 339:** line $-13$, "page 337" should be "pp. 337–338".

**Page 342:** line 4. "some fractions" should be "some nonzero fractions."

**Page 344:** lines $-8$ and $-7$.

> (see the introduction of Chapter 17) and in the definition of

should be

> (see the initial paragraph of section 17.1) and in the discussion of

**Page 344:** line $-6$. "page 292." should be "page 295."

**Page 348:** replace line $-1$ by:

$$\frac{20 \text{ quarts}}{60 \text{ seconds}} = \frac{20}{60} \text{ quarts/second} = \frac{1}{3} \text{ quarts/second}$$

**Page 349:** line 7. "$85\frac{1}{5}$" should be "$85\frac{1}{15}$".

**Page 350:** the last sentence of footnote 6:
"It something is true" should be "If something is true"

**Page 350–351:** replace the passage starting from the second paragraph, line 12 on page 350 ("We are going to show ...") to line 7 on page 351 ("...is $5\frac{5}{11}$ hours.") by the following simpler argument.

Because Regina mows at a constant rate, if she mows $B$ sq. ft. in $t$ hours, then $\frac{B}{t} = R$, so that $B = Rt$ sq. ft. Thus the amount of lawn mowed by Regina in $t$ hours is $Rt$ sq. ft. Similarly the amount of lawn mowed by Eric in $t$ hours is $Et$ sq. ft. The amount of lawn mowed by Regina and Eric together in $t$ hours is therefore

$$(R+E)\,t \quad \text{sq. ft.}$$

Suppose now Regina and Eric together finish mowing the lawn in $T$ hours. Then in $T$ hours they mow $A$ sq. ft. Hence $(R+E)\,T = A$. But we already know that $R = \frac{A}{10}$ and $E = \frac{A}{12}$, so

$$\left(\frac{A}{10} + \frac{A}{12}\right) T \;=\; A$$

It follows that

$$T \;=\; \frac{A}{\frac{A}{10} + \frac{A}{12}} \;=\; \frac{1}{\frac{1}{10} + \frac{1}{12}} \qquad \text{(by (b) on page 310)}$$

$$=\; \frac{1}{\frac{11}{60}} \;=\; 5\frac{5}{11}$$

So the time it takes both of them to do it together is $5\frac{5}{11}$ hours.

**Page 352:** replace line 2 to line 13, "saw above that ... Therefore," by the following:

saw above that $R = \frac{A}{10}$ sq. ft./hr. In other words, she mows $\left(\frac{A}{10}\right) t$ sq. ft. in $t$ hours. Similarly, Eric mows $\left(\frac{A}{12}\right) t$ sq. ft. in $t$ hours. Together, Regina and Eric mow

$$\left(\frac{A}{10}\right) t + \left(\frac{A}{12}\right) t = \left(\frac{1}{10} + \frac{1}{12}\right) (A \times t) = \left(\frac{11}{60}\right)(A \times t) \text{ sq. ft.}$$

in $t$ hours. In particular, they mow $\left(\frac{11}{60}\right) A$ sq. ft. in one hour. Now suppose the two together can finish mowing the lawn in $T$ hours. This means they mow $A$ sq. ft. in $T$ hours, so that $\left(\frac{11}{60}\right)(A \times T) = A$. Therefore,

**Page 353:** line $-5$. "16 HOURS" should be "12 HOURS".

**Page 354:** line 2. "$\frac{H}{16}$" should be "$\frac{H}{12}$".

**Page 354:** replace the passage from line 4 ("exactly as in ...") to the end of the page by the following:

exactly as in the preceding Problem 6, we conclude that, if they paint together, they would paint $(J + L + M)t$ sq. ft. in $t$ hours for any number $t \geq 0$. Now, using the values of $J$, $L$, and $M$ above, we see that, working together, they'd paint

$$\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) (H \times t) \text{ sq. ft. in } t \text{ hours.}$$

If they finish painting the whole house ($H$ sq. ft.) in $T$ hours, then

$$\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) (H \times T) = H.$$

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Since $\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) = \frac{37}{180}$, we get,

$$T = \frac{1}{\frac{37}{180}} = 4\frac{32}{37} \quad \text{hours.}$$

The answer is therefore 4 and $\frac{32}{37}$ hours, or approximately 4 hours and 52 minutes.

**Page 355:** ($i$) Replace the second and third lines of Exercise 1 by:

proved that (22.1) holds. Now prove the converse of (22.1), i.e., if $\frac{A}{\ell} = \frac{B}{m} = \frac{C}{n}$, then $A : B : C = \ell : m : n$.

($ii$) Line $-2$. Replace "flow is constant," by "flow, with or without the water-saving device, is constant,".

**Page 356:** Exercise 12, first line. Replace "317. Divide" by "317 about dividing". Second line: "Fourth" should be "fourth".

**Page 358:** line $-13$ (not counting the footnote). Change the period at the end of the line to a comma.

**Page 359:** line 6. "on the way and" should be "on the way out and".

**Page 361:** line 9 (not counting the picture). "look a the" should be "look at the"

**Page 364:** there are several corrections. ($i$) Replace the last sentence of Exercise 3 by:

What percentage increase in the number of women would make this reduction possible?

($ii$) Exercise 4. Second line: "to reduce" should be "that reduces". Last sentence in parentheses should read:

> (Assume that the rate of water flow, with or without the water-saving device, is constant.)

($iii$) Exercise 5, first line. Replace

> "100%" by volume

by

> "100% by volume"

**Page 365:** ($i$) line 1. "alcohol is 70% of the liquid" should be "alcohol is 70% of the liquid by volume".

($ii$) Line 2. "50% alcohol" should be "50% rubbing alcohol".

**Page 370:** the first full paragraph near the middle of the page. "Sometimes one could" should be "Sometimes one can"

**Page 372:** ($i$) Line 14. "the former problem" should be "the latter problem".

($ii$) Line 21. "hopes" should be "would hope".

**Page 376:** the first line below the third number-line picture. "the symbols $x < y$ have" should be

> the symbol "$<$" in $x < y$ has"

**Page 381:** second line of the second paragraph. "the rational numbers on the right of 0" should be "the rational numbers that are 0 and to the right of 0".

**Page 382:** ($i$) Lines 3 to 6 of section 27.1.

Specifically, it needs the concept ...as an **endpoint.**

should be

> Now, when we define the addition of two rational numbers, *we may assume that both numbers are nonzero* because, by definition, 0 added to any number $x$ is just $x$ itself. With this understood, the definition of rational number addition needs the concept of a **nonzero vector**, which is, by definition, a segment on the number line together with a designation of one of its two endpoints as its **starting point** and the other as its **endpoint**.

($ii$) Line 11 of Section 27.1. "We denote vectors" should be

> Unless stated to the contrary, a *vector* will refer to a *nonzero vector*. On rare occasions, we will need the concept of the **zero vector**, which has zero length and no direction. We denote (nonzero) vectors

**Page 384:** Replace lines 5-7 by:

> Equally clearly, $\vec{x} + \vec{0} = \vec{0} + \vec{x} = \vec{x}$ for every $x$.
> The following observation is critical in dealing with the addition of special vectors $\vec{x} + \vec{y}$. The case where $\vec{y}$ is equal in length but opposite in direction to $\vec{x}$ (i.e., $y = x^*$) has just been dealt with: $\vec{x} + \vec{x^*} = \vec{0}$. Thus we may assume in the following observation that $y \neq x^*$.

> **Key Observation.** *Let $x$ and $y$ be nonzero rational numbers so that $y \neq x^*$. Then*

(i) *the direction of the sum $\vec{x} + \vec{y}$ is the common direction of the vectors when they have the same direction, and is the direction of the vector whose length is greater when they have opposite directions, and*

**Page 384:** line $-6$ (not counting the footnote and not counting the number lines). "shorter than $y$," *should be* 'shorter than $\vec{y}$,"

**Page 385:** line $-11$ (not counting the number lines). "also follows from" should be "can also be achieved by".

**Page 386:** ($i$) Line 9. "$\vec{z} = \vec{x} + \vec{y}$. However, if" should be "$\vec{z} = \vec{x} + \vec{y}$. As usual we can skip the trivial case of $y = x^*$. So if".

($ii$) Line $-10$. Delete line $-10$ (not counting the number-line picture): "the endpoint ... In other words,".

**Page 387:** there are several corrections.
($i$) Line 1. "By (ii) of the Key Observation," *should be* "By the definition of rational number addition and the observation above section 27.3,".

($ii$) The end of line 11. "as one of" should be "as at least one of".

($iii$) Line 17. "and (vii) above," should be "and the definition of the addition of rational numbers".

($iv$) line 19. "*but if they are negative*" should be "*and if they are both negative*".

**Page 388:** there are several corrections.

($i$) Lines 1. "(vi)" should be "(viii)".

($ii$) Lines 3 and 4. "picture drawing" should be "picture-drawing".

($iii$) Line 5. Delete "and $2^* + 5 = (2 + 5^*)^*$".

($iv$) Line 10. Delete "This also follows from (vi)."

($v$) Line $-5$. "(iv) and (v) on page 385" should be "(i) and (ii) on page 384".

**Page 389:** several corrections.

($i$) Second number-line picture. The notch to the immediate right of the number 1 should be labeled as "$\frac{7}{6}$".

($ii$) Line $-5$. "$\overrightarrow{(t-s)}$" should be "$\overrightarrow{(t-s)^*}$"

($iii$) Line $-1$. "$(\frac{8}{7})^* + \frac{1}{3}$" should be "$(\frac{1}{3}) + (\frac{8}{7})^*$".

**Page 390:** ($i$) Line $-12$. "$x > y$" should be "$x \geq y$".

($ii$) Line $-11$. "in Part 2" should be "in section 16.1".

**Page 391:** ($i$) Line 3. "addition" should be "subtraction".

($ii$) Line 5. "we abandon the notation" should be "we will generically abandon the notation".

**Page 392:** line $-13$. "Using (27.8) twice in succession" should be "Using (27.1) twice in succession".

**Page 394:** Exercise 5, first line. "then $x = 0$" should be
"then $y = 0$"

**Page 397:** line $-18$. "proper place" should be
"proper perspective".

**Page 398:** lines 12 and 13. Delete "on account of the uniqueness assumption on the sum of two numbers in (A1)."

**Page 399:** line 7. "positive" should be "nonzero".

**Page 402:** Exercise 3 should be moved to the Exercises of Chapter 29 on page 412 (because this exercise requires that we know how to multiply rational numbers).

**Page 404:** line 5. "This sections" should be "The sections"

**Page 404:** line $-4$ (not counting footnote).
"(by (M2)" should be "(by (M2))"

**Page 406:** (*i*) Replace lines 14–15, i.e., the following two lines

$$\begin{aligned} &= \ 0 \cdot (-1) \qquad\qquad \text{(by (M3))} \\ &= \ 0. \end{aligned}$$

should be

$$\begin{aligned} &= \ 0 \cdot (-1) \\ &= \ 0, \qquad\quad \text{(by (M3))}. \end{aligned}$$

(*ii*) Line $-10$ (not counting the footnote). "(−1), and the uniqueness statement in (M1) says" should be "(−1) to get that".

**Page 409:** ($i$) Line 10. "is no different" should be "is similar".

($ii$) Footnote. "page 397" should be "page 398".

**Page 413:** line $-7$.
"When $y > 0$" should be "When $x > 0$ and $y > 0$".

**Page 414:** line 4. "Basic Fact 4" should be "Basic Fact 5"

**Page 415:** line $-14$. "$x,\ y\ z$" should be "$x,\ y$".

**Page 421:** line $-2$. "In general, if $x < 0 < y$," should be "If $x$ or $y$ is 0, this assertion is trivial. So we may assume both $x$ and $y$ are nonzero. Suppose $x < 0 < y$,".

**Page 422:** there are several corrections.
($i$) Line $-11$. "$-(x - y) < 0$" should be "$-(x - y) > 0$".

($ii$) Line $-8$. "It should be remarked that sometimes (C) is taken" should be "It should be remarked that, in mathematics, sometimes (C) is taken".

($iii$) Line $-5$. "the fact that $z > 0$ means" should be "the significance of $z > 0$ is that".

**Page 423:** ($i$) Line 8. "we have $(y - x)z > 0$" should be "the definition of the multiplication of fractions implies $(y - x)z > 0$".

($ii$) Line 11. "and $x < y$, if $x < 0 < y$" should be "and $x < y$. If one of $x$ and $y$ is 0, (D) is trivial. We may therefore assume that both $x$ and $y$ are nonzero. If $x < 0 < y$".

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**Page 424:** there are several corrections.

(*i*) Line $-5$. "to (E)," should be "to (D),".

(*ii*) Line $-4$. "formula" should be "formulas".

(*iii*) Line $-2$. "By (E)," should be "By (D),".

**Page 425:** (*i*) Line 6. "$x^0 = 1$" should be "$\boldsymbol{x^0 = 1}$". (boldface)

(*ii*) Line 11. "Note that" should be "We claim:".

**Page 426:** There are many corrections.

(*i*) Line 3. "$1 > x^3,\ 1 > x^3$" should be "$1 > x^3,\ 1 > x^4$"

(*ii*) The end of line 10. "(A)" should be "(D)".

(*iii*) Second line of section 31.3. "$|\boldsymbol{x}|$ of a number $x$" should be "$|\boldsymbol{x}|$ of a nonzero number $x$".

(*iv*) Fourth line of section 31.3. "In particular," should be "We also define $|\boldsymbol{0}|$ to be 0. Thus".

(*v*) Line $-9$. Replace

> is equivalent to the two inequalities $-b < x \leq 0$ and
> $0 \leq x < b$,

by

> is equivalent to $x$ not being equal to $-b$ or to the left of $-b$,
> and not being equal to $b$ or to the right of $b$,

(*vi*) Line $-5$. " [a,b]" should be " $[-b, b]$".

($vii$) line $-3$. "$x \in (-b, b)$"  should be  "$x$ lies in $(-b, b)$".

**Page 427:** line 2. "negative"  should be  "$\leq 0$".

**Page 429:** Exercise 2, line 2.  Delete  "(cf. Example 4 on page 243)".

**Page 433:** first sentence of the second paragraph.
"Among the highlights of this chapter" should be
"Among the highlights of Part 4"

**Page 435:** line $-1$. "Chapter 7"  should be  "section 7.1".

**Page 437:** line 7. "whole number"  should be  "integer".

**Page 438:** line $-8$. "either: $\pm 2A$"  should be  "either: $-A$, $\pm 2A$".

**Page 440:** ($i$) Line $-7$. "$6 + 4 + 7$"  should be  "$6 + 7 + 4$".

($ii$) Line $-6$. "$84 = 26 \times 3$"  should be  "$84 = 28 \times 3$".

**Page 441:** line $-2$. "Lemma 32.1. If 572"  should be:

Lemma 32.1. Since we know 4 does not divide 86, 4 does not
divide 93,386 either, and vice versa. If 572

**Page 442:** line $-2$. "1(b)"  should be  "1(a)".

**Page 443:** line 4. "$k \geq 2$"  should be  "$k \geq 1$"

**Page 444:** ($i$) Exercise 1. The order of (a) and (b) should be reversed. In other words, the first part of the exercise should be:

> (a) Prove that every odd number is of the form $2n+1$, where $n$ is a whole number. (b) Prove that the product of two odd numbers is odd, the product of two even numbers is even, and the product of an odd number and an even number is even.

($ii$) Exercise 3. "59143" should be "59103".

**Page 445:** second line of section 33.1. "If $b|a$ but" should be "If $b$ is a whole number and $b|a$ but".

**Page 446:** there are several corrections.
($i$) Line 7. "*neither a composite nor a prime.*" should be "*neither a composite nor a prime.* The reason for excluding 1 from being a prime is to ensure that the uniqueness part of the fundamental theorem of arithmetic on page 457 is valid."

($ii$) Line 19: "*can divide* 97" should be "*can be a proper divisor* of 97".

($iii$) Line $-18$: "a divisor of 97" should be "a proper divisor of 97".

($iv$) Line $-16$: "*a divisor of n.*" should be "*a proper divisor of n.*"

($v$) Line $-11$: "no divisors" should be "no proper divisors".

**Page 447:** there are several corrections.
($i$) 4th line of second paragraph. "Since each of $b$ and $c$ is a divisor of 193, it" should be "Since each of $b$ and $c$ is a proper divisor of 193, each".

($ii$) 4th line of third paragraph. "each of $b$ and $c$ is a divisor" should be "each of $b$ and $c$ is a proper divisor".

($iii$) Line $-5$ (not counting the footnotes). "in Chapter 17 we proved in general that" should be "by (D) on page 422,".

**Page 448:** ($i$) Line 4. "(see Chapter 16)" should be "(see page 240)".

($ii$) Line $-13$. Delete the sentence, "The proof of the lemma ... see page 50."

**Page 449:** there are several corrections.
($i$) Lines 8-9. Replace this indented passage by

*For a whole number $k$ satisfying $2 \leq k \leq 144$, it is a prime if and only if it has no proper divisor among the eleven numbers 2, 3, ... 11, 12.*

($ii$) Line 11. "Suppose we have" should be "If $k$ is a prime, naturally it has no proper divisors. Conversely, suppose we have".

($iii$) Line 14. "$2 \geq \ell \leq \sqrt{k}$" should be "$2 \leq \ell \leq \sqrt{k}$".

($iv$) End of line 18. "a divisor of $k$" should be "a proper divisor of $k$".

($v$) Line $-16$. Replace "*whole numbers $\leq 144$, ... be divisible by*" by

*whole numbers $\geq 2$ and $\leq 144$, what remains cannot be proper multiples of*

**Page 450:** There are three lists of certain whole numbers up to 144, and there are errors in the first two lists. In both lists of numbers, 135 and 141 should be deleted, and 133 and 137 should be added.

**Page 451:** there are several corrections.

($i$) End of line 13. "58711 digits" should be "58711 digits in each prime".

($ii$) End of line $-9$ (not counting the footnote). Replace "where" by "where $n \geq 1$ (because $k > 3$) and".

($iii$) Line $-2$ of the footnote. "beyond 4" should be "beyond 2".

($iv$) Footnote 3 at the bottom should read: "Two theorems proved in the 21st century by extremely difficult methods would serve to give you an idea. In 2013, Harald Helfgott proved Goldbach's weak conjecture, which states that every odd number $> 5$ is the sum of three prime numbers (`https://arxiv.org/abs/1501.05438`); this is an improvement on a theorem of I. M. Vinogradov, to the effect that any odd number bigger than a certain (very big) number is the sum of at most three primes. A second theorem states that there is a whole umber $k$, $k \leq 246$, so that if $p_n$ is the $n$-th prime number, there are an infinite number of $n$ so that $p_{n+1} - p_n = k$. Goldbach's conjecture is the assertion that this theorem is true with $k = 2$. The ground-breaking work in this direction is due to Yitang Zhang (2014), who proved the theorem for some $k \leq 7 \times 10^7$. Subsequently, J. Maynard (2015) improved the theorem to $k \leq 600$, and the theorem as stated for $k \leq 246$ is due to the Polymath Project8 (`https://asone.ai/polymath/index.php?title=Bounded_gaps_between_primes`)."

**Page 452:** bottom line. "$2 \times 10^{10}$" should be "$1.38 \times 10^{10}$".

**Page 453:** (*i*) Replace lines 8–10 by:

both primes, but 5040, 5041, 5048, 5049, and 5050 are not (the fact about 5041 is not obvious, see Exercise 7 below). So we get in fact a string of not 6 but 11 consecutive composites from 5040 to 5050 as a bonus. A more startling fact is that it is not at

(*ii*) Line 11. "9" should be "13".

**Page 454:** Exercise 4. "$n$ which is at most 20736" should be "$n > 1$ which is at most 21316 $(= 146^2)$".

**Page 457:** (*i*) Line 2. "**uniqueness**" should be "uniqueness". (no boldface)

(*ii*) Line 10. "*uniqueness of prime decomposition*" should be "**uniqueness of prime decomposition**".

**Page 459:** lines $-5$ to $-1$. Replace "is $< 13$. Indeed, if every one of them ... therefore $7 \times 13$" by

is $< 10$. Indeed, if every one of them is $\geq 10$, then their product is $\geq 10 \times 10 = 100 > 91$ so that it cannot be equal to 91. Thus we will try all the primes $< 10$ to see if any of them divides 91. Among 2, 3, 5, 7, only 7 does and the only possible prime decomposition of 91 is therefore $7 \times 13$.

**Page 460:** several corrections on this page.
(*i*) Line 6. "from 2 to $\frac{91}{2}$" should be "from 2 to 7".

(*ii*) Line 8. "primes from 2 to $\frac{n}{2}$" should be "primes from 2 to $\sqrt{n}$".

(*iii*) End of line 8. "500 digits" should be "2000 digits".

**Page 461:** Exercise 1. "smallest divisor" should be "smallest proper divisor".

**Page 463:** (*i*) Lines −7 and −6 (not counting the footnote) should be (the changes are indicated in red):

1, 2, 3, 4, 6, 8, 12, 24,
1, 2, 3, 6, 9, 18, 27, 54.

(*ii*) Line -4. "2, 6," should be "1, 2, 3, 6,".

**Page 464:** (*i*) Lines 1 and 2 should be (the changes are indicated in red):

1, 3, 7, 9, 11, 21, 33, 63, 77, 99, 231, 693,
1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

(*ii*) Line 4 should be "1, 3, 7, 21,".

**Page 465:** (*i*) Second sentence of the first full paragraph. "The is so" should be "This is so".

(*ii*) Line −17. "by iterating the process" should be "iterating the process"

**Page 468:** line −8. Replace "themselves. Let $a > b$." by

themselves. We may skip the trivial case where $a = b$. We will also skip the case where one of them, let us say $b$, is 0 because $\gcd(a, 0) = a$. Thus let $a > b > 0$.

**Page 469:** $(i)$ Lines 6 & 7. Replace "Now perform ... on $b$ and $r$,"
by

> Now if $r = 0$, then $\gcd(b, r) = b$ and therefore $\gcd(a, b) = b$.
> We are done in that case. So assume $r \neq 0$ and we repeat the
> division-with-remainder on $b$ and $r$,

$(ii)$ Line $-15$ should be "$r_1 = q_3 \boxed{r_2} + 0$,".

**Page 471:** there are several corrections.
$(i)$ Line 6. "by the Theorem 35.2" should be "by Theorem 35.2".

$(ii)$ Line 15. Replace "$p = q.$³ Then" by "$p = q$ but $p \neq s$". Then".

$(iii)$ Lines $-12$ & $-11$. Replace "the same argument if $p = s$ or
$q = s$." by

> similar arguments if $p = q = s$ so that $n = p^3$.

$(iv)$ Lines $-4$ & $-3$. Replace "A similar argument applies" by "Arguments similar to these apply".

$(v)$ Delete footnote 3.

**Page 474:** There are many corrections.
$(i)$ Line 5: The first sentence of the **Lemma** should be "Let $b$ and
$n$ be whole numbers $\geq 2$."

$(ii)$ Line 9. "$m$ and $n$" should be "$m$ and $n$ both $\geq 2$".

$(iii)$ Line 16: "There is related concept." should be
"There is a related concept."

($iv$) Line 17. "whole numbers $a$ and $b$ to be the smallest whole number $m$" should be

nonzero whole numbers $a$ and $b$ to be the smallest nonzero whole number $m$

($v$) Line $-13$. "multiples of of both numbers" should be "nonzero multiples of both numbers".

($vi$) Line $-11$. "108, 216, 270, 324, ..." should be "108, 216, 324, ..."

($vii$) Line $-3$. "product of all the primes in 24 and 108" should be "product of all the prime factors of 24 and 108".

**Page 475:** there are several corrections.
($i$) Lines 2-4. Delete the sentence, "Precisely, any prime ... 24 and 108."

($ii$) Line 12. "page 463" should be "page 464".

($iii$) In the listing of multiples of 210 in lines 14–16, insert the number 2310 between 2100 and 2520 on line 14.

($iv$) Line $-4$. "the primes common" should be "the copies of primes common".

**Page 476:** there are many corrections.
($i$) Line 4. The statement of **Proposition 1** should be

Let $a$ and $b$ be whole numbers $\geq 2$. If $d = \gcd(a, b)$, then $\gcd(\frac{a}{d}, \frac{b}{d}) = 1$.

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(*ii*) Lines 16 & 17. "another prime" should be "another copy of a prime".

(*iii*) Line 19. "*d* contains all the primes common to both *a* and *b*" should be "*d* contains all the prime factors common to both prime decompositions of *a* and *b*".

(*iv*) Line $-7$. "whole numbers $p$ and $q$" should be "integers $p$ and $q$".

(*v*) Line $-5$. "common divisor $c$" should be "common positive divisor $c$".

(*vi*) Line $-1$. "whole number" should be "nonzero whole number".

**Page 477:** (*i*) Line 1. "*whole number*" should be "*nonzero whole number*".

(*ii*) Line 10. "As before, let" should be "If one of $a$ and $b$ is 1, the proposition is trivial. So we may assume both $a$ and $b \geq 2$. Let
**Page 478:** there are several corrections.
(*i*) Lines 9 & 11. "common multiple" should be "nonzero common multiples". (*twice*)

(*ii*) The line above **Proposition 3**: "Therefore" should be "Since any nonzero number divides 0, we have, in general".

(*iii*) Line $-15$. "any common multiple" should be "any nonzero common multiple".

(*iv*) Line $-12$. "smallest number of the prime factors" should be "smallest number of copies of the prime factors".

($v$) Line $-11$. "other common multiples" should be "other nonzero common multiples".

**Page 479:** there are several corrections.
($i$) Line 10. "in Chapter 13" should be "(Theorem 13.1)".

($ii$) Line 15. "gcd$(c, d) = 1$" should be "gcd$(a, b) = 1$".

($iii$) Line 16. "(Chapter 13)" should be "(Theorem 13.2)".

($iv$) Lines 20 & 21. "Recall that we also have $aB = Ab$, so $b = B$" should be "Since $aB = Ab$, we get $B = b$".

($v$) Line $-5$. "$m$ and $k \Longleftrightarrow$ the prime" should be "$m$ and $k$, where $k > 0 \Longleftrightarrow b = 1$ or the prime".

**Page 480:** ($i$) Line 5. "Theorem 36.1" should be "the Lemma on page 474".

($ii$) Line 15. "By the lemma" should be "Either $b = 1$, or $b \geq 2$ so that by the lemma".

**Page 481:** there are several corrections.
($i$) Line 3. "*A whole number is*" should be "*A whole number* $\geq 2$ *is*".

($ii$) Line 6. "prime decomposition of $n$" should be "prime decomposition of $m$".

($iii$) The line above Theorem 36.4. Replace "First, we show something simpler." by

First, we show something simpler. Recall from page 446 that

we are assuming an advanced result, to the effect that every
positive number $x$ has a unique positive number $s$ (a point on
the number line to the right of 0) so that $s^2 = x$. This $s$ is
called the *positive square root of $x$* and is denoted by $\sqrt{x}$.

$(iv)$ Lines $-11$ & $-10$ (not counting footnote). Replace "until only
one of $a$ and $b$ is even and the other odd." by

> until at least one of $a$ and $b$ is odd.

**Page 482:** There are many errors.
$(i)$ Line 5. "Squaring both sides" should be

> If $b = 1$, then $\sqrt{3}$ would be equal to the whole number
> $a$, a contradiction. So $b \geq 2$, and by squaring

$(ii)$ Line 7. "of $b^2$" should be "of $b$".

$(iii)$ The right side of equation (36.1) should be "$3p_1^2 p_2^2 \cdots p_k^2$".

$(iv)$ 4th line below equation (36.1), at the very end of the line,
"$p_1 p_2 \cdots p_k$" should be "$p_1^2 p_2^2 \cdots p_k^2$".

$(v)$ 5th line below equation (36.1), Delete "according to Lemma
36.3".

$(vi)$ 7th line below equation (36.1),
"$3p_1 p_2 \cdots p_k$". should be "$3p_1^2 p_2^2 \cdots p_k^2$"

$(vii)$ 8th line below equation (36.1),
"$3p_1 p_2 \cdots p_k$". should be "$3p_1^2 p_2^2 \cdots p_k^2$"

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($viii$) 10th line below equation (36.1),
"$3p_1p_2 \cdots p_k$". should be "$3p_1^2\,p_2^2 \cdots p_k^2$".

($ix$) Three lines above equation (36.2). "Then $p = \frac{a^2}{b^2}$" should be

> If $b = 1$, then $\sqrt{p}$ would be equal to the whole number
> $a$, a contradiction. So $b \geq 2$. Then $p = \frac{a^2}{b^2}$

($x$) Two lines above equation (36.2). "$b^2$" should be "$b$".

($xi$) Line $-7$. Replace "the prime decomposition $q_1q_2 \cdots q_k$" by
"the prime decomposition $q_1^2\,q_2^2 \cdots q_k^2$".

($xii$) Line $-7$ to line $-6$, Delete "according to Lemma 36.3".

($xiii$) Line $-5$. "$pq_1q_2 \cdots q_k$" should be "$pq_1^2q_2^2 \cdots q_k^2$"

($xiv$) Line $-4$. Replace "But $3b^2 = a^2$, so by FTA, $pq_1q_2 \cdots q_k$" by

> But $pb^2 = a^2$, so by FTA and equation (36.2), $p\,q_1^2\,q_2^2 \cdots q_k^2$

($xv$) Line $-1$. Replace "in $pq_1q_2 \cdots q_k$" by

> in $p\,q_1^2\,q_2^2 \cdots q_k^2$, which is the right side of (36.2),

**Page 483:** there are several corrections.
($i$) Line 17. Replace "Putting these prime decompositions" by

> Substituting these expressions of $a^2$ and $(5b)^2$ as
> products of primes

($ii$) Line $-11$. "there is" should be "there are".

($iii$) Line $-4$.  "by definition"  should be  "by the definition of $N$".

**Page 484:** line 2. "there is"  should be  "there are".

**Page 487:** line $-1$.  "$k$ whole numbers"  should be
  "$k$ whole numbers not all zero".

**Page 498:** ($i$) Insert  the following paragraph between line 6 and line 7 (immediately above the **Pedagogical Comments**:

> Recall our *convention* (page 188) that, unless stated otherwise, a "decimal" will be understood to be a "finite decimal".

($ii$) Line 18.  "*finite decimal with a nonzero decimal part*"  should be  "*finite decimal*".

**Page 499:** line 10. "that that"  should be  "that".

**Page 500:** ($i$) Two lines below equation (39.1).  "$m$ and $n$ are positive"  should be  "$m$ and $n$ are nonnegative".

($ii$) Three lines below equation (39.1).  "$m > 0$"  should be  "$m \geq 0$", and "$m < 0$"  should be  "$m \leq 0$".

**Page 501:** line $-6$. "pull down the decimal"  should be  "pull down the decimal point".

**Page 504:** Exercise 2.  Insert  "Compute:"  at the beginning, and delete  all the "$= ?$".

**Page 506:** ($i$) Line $-12$.  "$76 < 79$"  should be  "$77 < 79$".

($ii$) Line $-3$.  "*Suppose two finite decimals*"  should be
"*Suppose two unequal finite decimals*".

**Page 507:** ($i$) Line 10.  "$A$ and $B$"  should be  "two decimals $A$ and $B$ with at most $k$ decimal digits".

($ii$) Line 13.  "$B$ has"  should be  "$B \times 10^k$ has".

**Page 508:** line 6.  "0.000724"  should be  "0.0007624".

**Page 511:** ($i$) Line 8.  "Review of Division-with-Remainder"  *should be*  "The Complete Expanded Form of a Finite Decimal".

($ii$) Line 11.  The title of section 4.11  should be:
**The Complete Expanded Form of a Finite Decimal**

**Page 512:** lines 12 & 13.  Replace  the sentence  "The exponent of 10 ...of the digit."  by

> In the complete expanded form of a finite decimal, the power of 10, $10^m$, of $k \times 10^m$ (where $k$ is a single-digit whole number) is called the **generalized place value** of the digit $k$.

**Page 513:** ($i$) Line 12.  "by definition the number"  should be "equal to".

($ii$) 4th and 3rd lines above displayed equations (41.1). "adding an infinite number of whole numbers"  should be "adding an infinite number of fractions".

**Page 514:** ($i$) Line 7.  "gets closer and closer to 0 as $n \rightarrow \infty$" should be  "goes to 0 as $n \rightarrow \infty$".

($ii$) Third line of Theorem 41.1. "*positive*" should be "*nonnegative*".

**Page 515:** lines $-15$ & $-14$. "and finally, the definition of when two numbers are equal" should be "so that finally we can make sense of $0.9999999\cdots = 1$".

**Page 517:** 4th line of Theorem 41.2. "*positive or negative*" should be "*nonnegative or negative*".

**Page 518:** ($i$) Line 1. "*positive*" should be "*nonnegative*".

($ii$) Line 16. "using the Lemma" should be "using Lemma".

**Page 521:** replace line $-1$ by

it is equal to a finite decimal whose last (rightmost) decimal digit is 5, or if it ends in $7\overline{9}$, then it is equal to a finite decimal whose last decimal digit is 8, etc. For

**Page 523:** in exercise 5, the order of (d) and (e) should be interchanged. In other words, it should be:

(d) $0.\overline{142857}$, (e) $1.\overline{285714}$,

**Page 525:** third line of Theorem 42.1. "$m \times 10^k$ by $n$, where $k$ is any whole number $\geq n$" should be "$m \times 10^c$ by $n$, where $c$ is any whole number $\geq n$". (The change in notation from $k$ to $c$ is to avoid confusion in the proof of Theorem 42.1 on pp. 528 ff., where the letter $k$ is used for a different purpose.)

**Page 526:** ($i$) As a result of the change in the exponent from $k$ to $c$ in Theorem 42.1 on page 525, the exponents "$k$" on lines 1, 3, 4, and

8 (twice) should be changed to the exponent "$c$".

($ii$) **Delete** the **Activity**. (It cannot be done without an additional piece of information about the "almost uniqueness" of the decimal that is equal to a given number.)

**Page 527:** line $-3$.  "$\frac{1818}{1000}$"  should be  "$\frac{1818}{10000}$"

**Page 528:** line $-2$.  "For definiteness,"  should be  "Since the two cases are similar, for definiteness,"

**Page 529:** ($i$) **Add** to the end of **Remark** (1) in middle of page:

> Thus the whole number $k$ can be made as large as we like and the result does not change. This partly explains the choice of the whole number $c$ in the statement of Theorem 42.1 to be larger than perhaps strictly necessary (it doesn't matter, as we have just seen). The other reason why $c$ is chosen to be so large is that in the *repeating case* of the theorem (see pp. 530 ff.), $c$ may have to be this large in order to get a repeating block of the decimal.

($ii$) **Remark** (3), line 2.  "$\frac{q}{10^k}$"  should be  "$\frac{m \times 5^k}{10^k}$".

**Page 530:** line 6 of the **Remark**.  The first two  "$<$"  should be  "$\leq$" . In other words, this line  should read:

> Thus $7q_n = (3 \times 10^n) - r_n \leq (3 \times 10^n)$, so that
> $q_n \leq \frac{3}{7} \times 10^n < 1 \times 10^n = 10^n$,

**Page 532:** ($i$) Line 5 (not counting the number-line picture). Replace "so that" by

and since we have just shown that $\frac{3}{7} = s_n + \left(\frac{r_n}{7} \times \frac{1}{10^n}\right)$, we have

$(ii)$ Line 9. "$\frac{3}{7} \geq s_n$" should be "$\frac{3}{7} - s_n \geq 0$".

**Page 533:** line 2 after the long division. "in boldface italic fonts" should be "in a boldface italic font".

**Page 535:** there are several corrections.
$(i)$ Line 3. "$3 \times 10^{108}$ (or $3 \times 10^k$ for any $k > 107$)" should be "$2 \times 10^{108}$ (or $2 \times 10^k$ for any $k > 107$)".

$(ii)$ Lines 15–16. "by 7 above (42.2)" should be "on page 533".

$(iii)$ Line $-3$. "$10^n - r_n < 10^n$" should be "$10^n - r_n \leq 10^n$".

$(iv)$ Line $-1$ should be (the changes are indicated in red):

$$q_n \leq \frac{1}{28} \times 10^n < 1 \times 10^n = 10^n,$$

**Page 536:** First line to the right of the first long division. "$1 \times 10^3 = (35 \times 28) + 2$" should be "$1 \times 10^3 = (35 \times 28) + 20$".

**Page 538:** line 13. "gets closer and closer" should be "goes".

**Page 539:** Exercise 7, second and third lines. "any whole number" should be "any nonzero whole number".

**Page 547:** left column. Above "division of finite decimals" (between line $-15$ and line $-14$), insert:

division by zero, 132–133

**Page 547:** right column. Line $-5$.
"reduced, **205**" should be "reduced, **206**"

**Page 548:** insert right below line $-12$ (as the last of the items beginning with "L", right below "117–119")

lowest terms (fraction), **206**

**Page 549:** left column. Above "number" (between line $-3$ and line $-2$), insert

NSF-EHR, 367

**Page 550:** left column. Insert between line $-4$ and line $-3$ (above "reducing fractions"):

reduced fraction, **206**

**Page 551:** ($i$) Left column. "transitivity, **25**, 239" should be "transitivity, **25**, 240".

($ii$) Right column. "Williams, E. M., 22" should be "Willingham, D. T., 41".