Math 55 Lecture 5 (Finish §2.3), §2.4, 2.5

Recall: Let A, B be nonempty sets. A function f from A to B is

Def: A function f is one-to-one or injective iff

Ex: f: Students in Math 55 $\to$ Cities defined by $f(x)$ = city where x went to high school.

Def: A function $f: A \to B$ is onto or surjective iff

Def: f is a bijection or one-to-one correspondence

Ex: Determine whether each function below is one-to-one and/or onto, considered as function from $\mathbb{R}$ to $\mathbb{R}$

(a)

(b)

(c)

(d)

Ex: Let A be set. The identity function $i_A : A \to A$ is the function

Def: Let f be one-to-one correspondence $f: A \to B$. The inverse function of f, denoted $f^{-1}: B \to A$, is

Def: Let $g: A \to B$ and $f: B \to C$ be functions. The composition of f and g, denoted $f \circ g$,

§ 2.4

Def: A sequence is

Ex 1: A geometric progression is the sequence of the form

Ex 2: An arithmetic progression is sequence of the form

Example: Find formula for sequence whose first terms are

Summation: We denote $a_m + a_{m+1} + \ldots + a_n$ by

Sums of geometric progressions often arise.

Theorem: If a, r real numbers, and $r \neq 0$, then

$$\sum_{j=0}^{n} ar^j =$$

Proof:

Def: Two sets A and B have same cardinality
iff

Def: A set that is either finite or has the same cardinality as the positive integers is called

Ex: The set of all integers is countable
Pf:

Theorem: The set of positive rational numbers is countable.

Pf:

Theorem: The set of real numbers is uncountable.

Proof: Use proof by contradiction.