Math 55 Lecture 3 §1.6, 1.7, 1.8

Universal instantiation:

Given the premise $\forall x\, P(x)$,
we can conclude that

Ex:

Rule of Inference	Name

Ex: Show that premises

Let $S_n(x)$ denote

let $A_s(x)$ denote

The domain is

Step	Reason
1.	
2.	
3.	
4.	
5.	

§1.7 Introduction to Proofs

A proof is a valid argument that establishes the truth of a mathematical argument.

A theorem is

A proposition is

A lemma is

A corollary is

A conjecture is

1. <u>Direct proof</u> of $p \Rightarrow q$.

<u>Rk</u>: To prove theorem of form $\forall x\ (P(x) \to Q(x))$, our goal is to

<u>Def</u>: The integer n is <u>even</u> if there

n is <u>odd</u> if

<u>Ex</u>: Show that the sum of two odd integers is even.

<u>Proof</u>:

2. Proof by contraposition:

Ex: Let $n \in \mathbb{Z}$ (the integers). Show that if n^2 is even then n is even.

Proof:

Note: Direct proof is harder, here...

3. Proof by contradiction:

Def: A real number r is rational if

Ex: Prove that $\sqrt{2}$ is irrational via ↯.

Proof:

Ex: Show that at least 10 of any 64 days chosen must fall on the same day of the week.

Pf:

Proofs of equivalence:
To prove a theorem that is a biconditional statement, i.e. $p \leftrightarrow q$, we show

Rk: iff shorthand for "if and only if"

Ex:

Pf:

Proof by Cases: Sometimes it is helpful to divide up proof into cases ...

Ex:

Ex:

Q: What cases should we consider?

Without loss of generality (WLOG):

Ex:

Pf:

Open Problems:

Ex:

Example: