Math 55 Lecture 25 §10.5

Def: An Euler circuit in graph G is

Ex: $G =$

Make graph:

Question becomes:

$G =$

Note: (1) If v is the start/end vertex, then it must have

(2) If v is not the start/end vertex, then for

Note: We always assume graphs have

Theorem: A connected graph G with at least 2 vertices has an Euler circuit iff

Example: Do these graphs have Eulerian circuit?

Lemma: Any multigraph G w/ at least 2 vertices, s.t. all degrees are even, has

Proof:

Proof of Theorem: We already proved that if G has E. circuit,

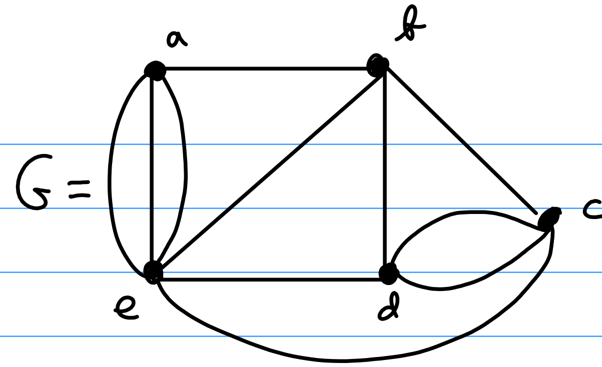
Need to show:

Step 1: Use Lemma to construct

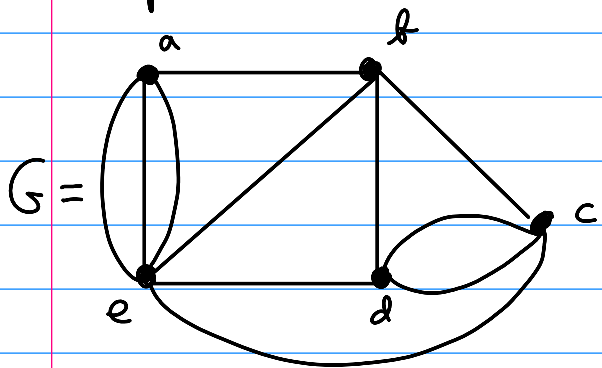
Step 2:

Step 3: Remove edges of C_1 from H_1, get subgraph H_2.

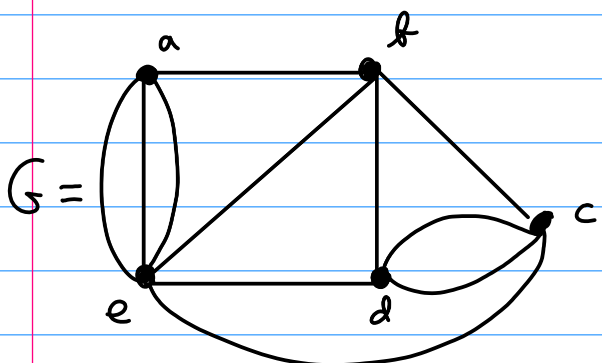
Ex: Does G have Eulerian circuit?



Step 1: Find a simple circuit C_0.

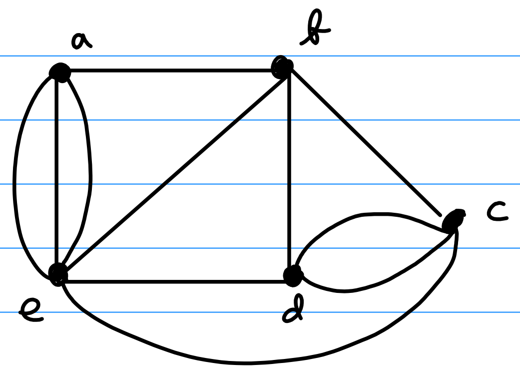


Step 2: Let H_1 be graph of unused edges of G.

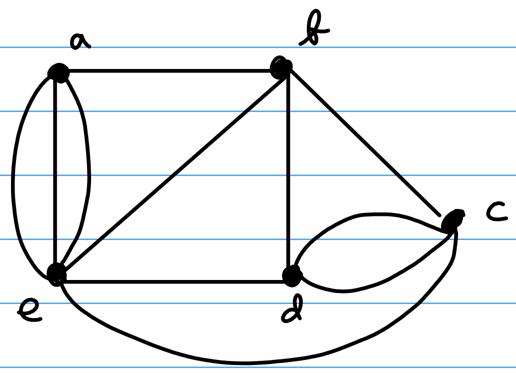


Step 3: Let H_2 be graph of unused edges.

$G =$



$G =$



Related concept: Euler path

Def: An Euler path in G is

Thm: A connected graph has an Euler path but not an Euler circuit iff

Ex:

Note: