Math 55 Lecture 24 § 10.3, 10.4

What's a good way to represent a graph?
Listing all vertices & edges is cumbersome

Def: Let $G=(V,E)$ be an undirected graph w/ $|V|=n$. Denote vertices by $v_1,\dots,v_n$. The adjacency matrix A

Ex:

Ex:

Ex: Draw a graph w/ adj matrix

Obs: An adj. matrix of undirected graph is

Def: If $G = (V, E)$ is a directed graph, its adj. matrix A (or A_G) is the $n \times n$ matrix s.t.

Ex:

Not in general a

Another way to represent graph:

Def: Let $G = (V, E)$ be undirected graph. Let $v_1, \ldots, v_n$ be vertices and $e_1, \ldots, e_m$ be edges. The **incidence matrix**

Ex:

What does it mean for two graphs to be the "same"?

Def: The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if

Ex: Are these 2 graphs isomorphic?

Note: If G_1 and G_2 are isomorphic, they must have

If G_1 and G_2 are 2 graphs w/ n vertices, can be hard to determine whether they are isomorphic:

If we think 2 graphs not isomorphic, good strategy is to

Def: A property preserved by isomorphism is a

Ex: If G_1 and G_2 are isomorphic and G_1 has n vertices,

Ex: Show that these 2 graphs are not iso.

§ 10.4 Connectivity

A path in graph is

Def: Let $n \in \mathbb{N}$ and G an undirected graph. A path of length n from u to v is

If graph simple, can just give

Def: Path is circuit if

Def: Let $n \in \mathbb{N}$ and G a directed graph. A path of length n from u to v is

Ex of paths in graphs from real life.

Ex: Let $G = (V, E)$ where V = set of places, E =

Def: An undirected graph called connected if

Ex: In previous example, if V =

Ex: Which of these graphs is connected?

Recall: A subgraph of $G = (V, E)$ is a graph

Def: A **connected component** of a graph G is a

Ex: What are connected components of

$G =$

For directed graphs, 2 notions of connected:

Def: A directed graph is strongly connected if

Def: A directed graph is **weakly connected** if

Ex: Is G st. conn? weak conn?

$G =$

Paths / circuits can be helpful in determining questions of isomorphism.

Ex: If $f: G_1 \to G_2$ is a graph isomorphism, and

Are these 2 graphs iso?