Math 55 Lecture 22 §10.1, 10.2

1st review equiv. rel'n...

<u>Recall:</u> A relation R on set A is subset of $A \times A$.
It is an equiv. rel'n if it is:

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<u>Ex:</u> Let S be relation on $\mathbb{R}$ defined by
$S = \{ \qquad \}$
Show that S is equivalence relation.

<u>Reflex:</u> Need to show

<u>Symm:</u> Suppose $(x,y) \in S$. Need to show

<u>Trans:</u> Suppose $(x,y) \in S$ and $(y,z) \in S$.
Need to show

§10.1

<u>Def:</u> An (undirected) graph $G=(V,E)$ is

<u>Def:</u> If there are several edges between same 2 endpoints, called

<u>Def:</u> A <u>loop</u> is

Can use graphs to represent information.

<u>Ex:</u> Let V = set of all students in class.

Let $E=$

Part of graph might look like:

For some types of information, a

<u>Def:</u> A <u>directed graph</u> or <u>digraph</u> (V,E) is

Ex: Let $V =$ set of all species.
Draw edge u to v whenever

Ex: The web (internet). Let $V =$ set of all websites
Draw edge u to v if

Q: What if we want to model people
w/

Q: Same question but people w/

§10.2

Def: Two vertices u and v in undirected graph G
are

Def: The degree of vertex in undirected graph is

What do we get if we add all degrees of

Handshaking Theorem: Let $G = (V, E)$ be graph w/ e edges. Then

Pf:

"Handshaking" because:

Theorem: A graph has an even number of vertices of

Pf:

Special kinds of graphs:

The complete graph K_n

The cycle C_n has

The wheel W_n is

The n-cube Q_n

Def: A graph $G=(V,E)$ is bipartite if V can

,

Ex:

Thm: A simple graph is bipartite iff

§10.3

What's a good way to represent a graph?
Listing all vertices & edges is cumbersome

Def: Let $G=(V,E)$ be an undirected graph w/ $|V|=n$.
Denote vertices by $v_1, \ldots, v_n$. The adjacency matrix

Ex:

Ex:

Ex: Draw a graph w/ adj matrix $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Obs: An adj. matrix of undirected graph is symmetric, ie.