Math 55 Lecture 21 §9.1, 9.3, start 9.4 Relations

Def: Let A, B be sets. A **binary relation** R from A to B is

Notation:

Ex: Let A and B be two sets of cities.
Let $R \subseteq A \times B$ consist of

Ex: Let X and Y be sets, and $f: X \to Y$ a function.
Let $R \subseteq X \times Y$ consist of

Def: A **relation on the set A** is

Ex: Let A be the set $\{1,2,3,4\}$. Which ordered pairs are in the relation $R = \{(a,b) \mid a < b\}$?

Properties of Relations

Def: Relation R on set A called **reflexive**

Def: A relation R on a set A called **symmetric** if

Relation R called **antisymmetric** if

Ex: Let A be set of people.
Let $R = \{(a,b) \mid$
Reflexive?
Symmetric?
Antisymmetric?

Ex: Let A be a set of people.
Let $R = \{(a,b) \mid$
Is R reflexive?
Is R symmetric?
Is R antisymmetric?

Ex: A — set of people
$R = \{(a,b) \mid$
Reflex? Symm? Antisym?

Def: A relation R on set A is transitive if

Ex: Let $A = \mathbb{Z}^+$. Let $R = \{(a,b) \mid a,b \in A, a|b\}$.
Is R transitive?

Ex: Let $A = \mathbb{Z}$. Let $R = \{(a,b) \mid a,b \in A, a \equiv b \pmod 5\}$.

Is R reflexive?
Is R symm?

Is R trans?

Note:

New relations from old: 2 relations from A to B can be combined using set operations.

Ex: Let $A = \{1,2,3\}$, $B = \{1,2,3,4\}$.
Let $R_1 = \{(1,1),(3,2)\}$ and $R_2 = \{(3,1),(3,2),(3,4)\}$.
Then $R_1 \cup R_2 =$
$R_1 \cap R_2 =$

Def: Let R be relation from A to B and S a relation from B to C. The composite of R and S is

Ex: Let A be set of all people, let $R = \{(a,b) \mid a \text{ is child of } b\}$. What does $R \circ R$ represent?

Can represent relations by matrices & directed graphs.

To represent relation R from A to B by matrix M, make matrix with $|A|$ rows and $|B|$ columns.

Ex: Let $A = \{1, \ldots, 5\}$. Let $R = \{(a,b) \mid a \leq b\}$
Represent by matrix.

Def: A directed graph or digraph consists of

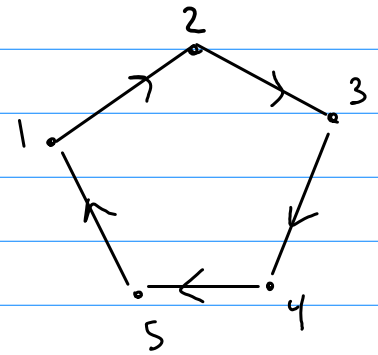
Ex:

Ex: Might be used to model transportation network

$A =$

$R =$

Ex: Let R be the relation on $\{1,2,3,4,5\}$ represented by



What is $R \circ R$?

Sol:

Start §9.4

Ex 1: Let A be set of places in Berkeley.

Rk:

Def: Let R be relation on set A. R may or may not have some property P (eg reflex). If there is relation S s.t.
•
•

Then S is P-closure of R.

Ex 2: Relation $R = \{(1,3),(2,2),(3,4)\}$ on set $\{1,2,3,4\}$ is not reflex.

$S = \{$ $\}$ — is reflex. closure of R.

In general: given relation R on A, can form reflexive closure of R by adding:

Ex 3: Use R of Ex 2. Not symm. What is its symm. closure?

In general: Can construct symm closure of R by taking union of R with $R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

Ex 4: What is symm. closure of R, where $R = \{(a,b) \mid a \text{ divides } b\}$ on set $\mathbb{Z}$. ?