Math 55 Lecture 2: Predicates + Quantifiers §1.4, 1.5, 1.6

Let $P(x)$ be the statement $x+2=10$.
Recall: Not a proposition, because

Called a <u>propositional</u> <u>function</u>.
Once we assign value to x,

Can also have statements involving several variables.
<u>Ex</u>: Let $Q(x,y)$ denote statement $x=y+3$

Another way to create a proposition from propositional function involves <u>quantifiers</u>...
English words used in quantifiers include

<u>Def</u>: The <u>domain</u> of a propositional function $P(x)$

<u>Def</u>: We use notation $\forall x\, P(x)$ to denote the statement

<u>Ex</u>: Let $P(x)$ be the statement "$x^2 < x$"
What is truth value of $\forall x\, P(x)$, where the domain consists)
How about if domain

"$\forall$" is the universal quantifier.

Def: We use notation $\exists x \; P(x)$ to denote the statement

Many other quantifiers besides $\forall$, $\exists$, such as:

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Def: The notation $\exists ! \; P(x)$ means

Example: Show that
$\exists x \; (P(x) \vee Q(x))$ and $\exists x \; P(x) \vee \exists x \; Q(x)$ are logically equivalent.

Need to show:

Solution: Must show 2 things:

(1)

(2)

How to negate a quantified expression:

How do we negate $\forall x\, P(x)$?

This illustrations logical equivalence:

Similarly,
These are <u>De Morgan's Laws for quantifiers.</u>

<u>Example:</u>

1.
2.
3.

Translated into math:

<u>Question:</u> Does #3 follow from #1 and #2?
(We'll learn how to determine, soon)

<u>§1.5</u> <u>Nested Quantifiers</u>

Two quantifiers are <u>nested</u> if one is in scope of another, e.g.

$$\forall x \ \exists y \ (x - 7y = 0)$$

Example: Let domain for variables x, y, z be real numbers $\mathbb{R}$. The statement

Ex: Let domain for x, y be $\mathbb{R}$. The statement

Note: Order of quantifiers matters!

Ex: Translate the statement
"The abs value of the product of 2 integers is the product of their absolute values."
Let domain be all integers $\mathbb{Z}$.

Ex of sudoku from last time:

$\bigwedge_{j=1}^{9} \bigwedge_{n=1}^{9} \bigvee_{i=1}^{9} p(i,j,n)$ expresses the statement
"every column contains each integer between 1 and 9."
How to write using quantifiers?

§1.6 Rules of Inference

Consider this argument:

1.
2.
3.

Is this a valid argument?

Def: An argument in propositional logic is . The final one is called & the rest called An argument is valid if

To verify that a statement is true, we use

To deduce new statements from statements we already have, we use

Example: Law of detachment or modus ponens is based on tautology:

Let p & q be the prop's
"My cell phone rings" and
"I will disturb the lecture."

(p.72) for bigger list

Rule of Inference	Tautology	Name

NB: Don't need to memorize these names!!

Ex: Use Rules of Inference to draw a conclusion based on the hypotheses

Let p, q, r be "I eat spicy foods,"
"I have strange dreams,"
"there is thunder while I sleep."
Then hypotheses are:

Step	Reason
1.	
2	
3.	
4.	
5.	
6.	

Conclusion:

Common mistake (fallacy):
The proposition $[(p \to q) \wedge q] \to p$ is not tautology.
(It is false when p is false & q is true).
Do not use it in an argument!

Rules of Inference for Quantified Statements

Universal instantiation:

Given the premise $\forall x\, P(x)$,
we can conclude that $P(c)$ is true
(provided c is element of domain).

Ex:

Rule of Inference	Name

Ex: