Math 55 Lecture 19

§8.4 Generating Functions
§8.5, 8.6 Inclusion-Exclusion

Def: The <u>generating function</u> for the sequence $a_0, a_1, \ldots, a_k, \ldots$ of real numbers is the infinite series

Ex: The gen. fun. for the sequence $1, 1, 1, 1, \ldots$ is

Note: $1 + x + x^2 + x^3 + \ldots =$

Ex: The gen. fun. for the sequence $1, a, a^2, a^3, \ldots$ is

Thm: Let $f(x) = \sum_{k=0}^{\infty} a_k x^k$ and $g(x) = \sum_{k=0}^{\infty} b_k x^k$.
Then $f(x) + g(x) =$

$f(x)g(x) =$

Ex: Let $g(x) =$

Sol:

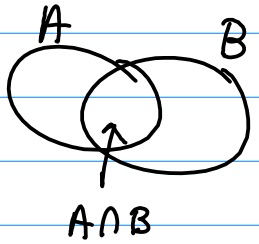
Try at home: Find the generating function for

Ex: Find the gen. function for the sequence $\{a_n\}$

Sol:

Inclusion-Exclusion

Saw earlier that if A, B sets, then $|A \cup B| =$



What if there are 3 sets?

Ex: 1232 students have taken a course in Spanish, 879 have taken course in French, 114 have taken course in Russian. 103 have taken classes in both S and F, 23 have taken courses in both S & R, 14 have taken courses in both F and R.

Sol:

Theorem (Inclusion-Exclusion): Let $A_1, A_2, \ldots, A_n$ be finite sets. Then $|A_1 \cup \ldots \cup A_n| =$

Pf:

Ex: How many solutions does $x_1 + x_2 + x_3 = 11$ have, where $x_1, x_2, x_3 \in \mathbb{Z}_{\geq 0}$ and

Sol: Let A = total # of solutions with $x_1, x_2, x_3 \in \mathbb{N}$.

Let $A_1 =$

$A_2 =$

$A_3 =$

Ex: Hatcheck Problem: New employee checks hats of n people at restaurant, forgetting to put claim check numbers on. Checker then gives back hats chosen at random from remaining hats.

Def: A permutation of numbers $\{1, 2, \ldots, n\}$ is a derangement if for all i,

Ex:

Thm: The number of derangements of set w/ n elements is $D_n =$

Pf: