Math 55 Lecture 18 §8.1, 8.2 Recurrence Relations

<u>Def:</u> A <u>recurrence relation</u> for the sequence $\{a_n\}$ is

<u>Ex:</u> Suppose that a person deposits \$10,000

<u>Sol:</u> Let P_n denote

Initial condition:

Ex: Tower of Hanoi.

Goal:

Rules:

Sol.

§8.2 Finding explicit solutions for linear recurrence relations

Def: A <u>linear homogeneous recurrence relation of degree k</u> w/ constant coefficient is

<u>Linear</u>:

<u>Homogeneous</u>:

<u>Constant coeffs</u>.

<u>Degree k</u>:

Ex: $f_n = f_{n-1} + f_{n-2}$

$a_n = a_{n-1} + a_{n-2}^2$

$H_2 = 2H_{n-1} + 1$

$B_n = n B_{n-1}$

Approach for finding solution of (1):

Note: $a_n = r^n$ is a solution

Def: We say $\overset{(2)}{r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k = 0}$ is the characteristic equation of $\overset{(1)}{a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}}$, and call

Theorem 1: Let $c_1, c_2 \in \mathbb{R}$. Suppose that $r^2 - c_1 r - c_2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ iff

Proof: 2 parts. ① Show that if r_1, r_2 are roots, and α_1, α_2 constants, then

② Show that if $\{a_n\}$ satisfies recurrence, then

Ex: Find explicit formula for

Sol:

Note: Theorem 1 doesn't work when char. equation has double root. In this case, use

Theorem 2: Let c_1 and $c_2 \in \mathbb{R}$, $c_2 \neq 0$. Suppose $r^2 - c_1 r - c_2 = 0$ has only one root r_0. A sequence $\{a_n\}$ is a solution of $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ iff

<u>Theorem 3</u>: Let $c_1, c_2, \ldots, c_k$ be real numbers. Suppose that the char. equation $r^k - c_1 r^{k-1} - \ldots - c_k = 0$ has k distinct roots, $r_1, r_2, \ldots, r_k$. Then a sequence $\{a_n\}$ is solution to recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$ iff

<u>Ex</u>: Find solution to

<u>Sol</u>: Char. poly is:

Factors: