Math 55 Lecture 17 §7.4

Def: A random variable is a function from

Note: Random variable is

Ex: Suppose coin

The expected value of a random variable is a

Def: The expected value (or expectation) of the random variable X on the sample space S is

Ex: What is the expected number of times a

$\therefore$

Notation: If X is a random variable on sample space S, let $\overline{p(X=r)}$ be

Lemma: If X a RV w/ range $X(S)$, $E(X)$

Redo Ex above:

Note: If f_1 and f_2 are functions from A to $\mathbb{R}$, we can

Def 1: $(f_1 + f_2)(x) :=$

$(f_1 f_2)(x) :=$

Def 2: Given $a, b \in \mathbb{R}$ and $f : A \to \mathbb{R}$, we can also define new function $(af+b) : A \to \mathbb{R}$ by

So if X_1 and X_2 are both random variables w/ sample space S, we can

And if $a, b \in \mathbb{R}$, get new RV $aX_1 + b$ defined by

Theorem "Expectation is linear": If X, X_i, $i = 1, 2, \dots, n$ are rand variables on S, and if a and $b \in \mathbb{R}$, then (i)

(ii)

Pf ($n=2$): (i) $E(X_1+X_2)$

(ii)

Ex: Suppose that n Bernoulli trials are performed, where p is the probability of success on each trial. What is

Sol:

Def: A random variable X has a geometric distribution w/ parameter p if

Ex: Suppose that the probability that a coin comes up tails is p.

Lemma:

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x} \text{ for } |x|<1 \text{ (limit of geom series formula)}$$

Differentiate both sides: $\displaystyle\sum_{j=1}^{\infty} j\, x^{j-1} = \frac{1}{(1-x)^2}$

Sol: What is the sample space?

Let X be random variable equal to

Theorem: If the random variable X has the geometric distribution w/ parameter p, then

Def: The random variables X and Y on a sample space S are independent if:

Theorem: If X and Y are indep random variables on a space S, then

Pf: $E(XY) =$

The expected value of a random variable tells us

What if we want to know how far from the average

Def: Let X be rand var on sample space S. The *variance* of X, denoted $V(X)$, is

$V(X) =$

Theorem: If X is rand var on sample space S, then

Using thm, can prove ...

Thm: If X and Y are two indep rand var's on sample space S,