Math 55 Lecture 14

§6.5 Generalized permutations & combinations

Ex 1: There are three little pigs who live in five (distinct) houses.
How many different ways can they arrange themselves in the 5 houses, if housemates are allowed? (even all 3 could be in same house)

Ex 2: How many different ways can they arrange themselves in the 5 houses, if there is at most one pig per house?

Ex 3: Now suppose that the pigs are interchangeable (all look the same - can't tell them apart).
How many ways are there of arranging 3 pigs in the houses, if there is at most one pig per house?

Ex 4: Again, suppose the pigs are interchangeable. How many ways are there of arranging 3 pigs in the houses, if housemates are allowed?

Method 1:

Method 2:

Another example of combination w/ repetition:

Ex: How many ways are there to select five bills from a banker, Sol :

Thm: There are
r-combinations
from a set w/ n element when repetition of elements is allowed.

Pf:

Ex: How many solutions does the equation have, where each $x_i \in \mathbb{N}$?

Sol:

Permutations w/ Indistinguishable objects:

Ex: How many different strings can be made by reordering letters of the word

Sol:

Thm: The number of different permutations of n objects, where there are n_1 indist. objects of type 1, n_2 indist. objects of type 2, ..., and n_k indist. objects of type k, is

Pf:

Dist. objects & dist. boxes

Ex: How many ways are there to give 5 cards to each of 4 players, from the standard deck of 52 cards? (all cards are distinct)

Thm: # of ways to distribute n distinguishable objects into k distinguishable boxes so that n_i objects are placed in box i, $1 \le i \le k$, is

More combinatorial proofs

Thm: Let $n, k \in \mathbb{N}$, w/ $k \le n$. Then

Pf (Combinatorial):