Math 55 Lecture 13: Permutations + Combinations, §6.3, 6.4

Def: A <u>permutation</u> of a set of distinct objects is

Ex: Let $S = \{1, 2, 3\}$. Then

Ex: How many permutations of S are there?

Ex: If $|S| = n$,

Def: An <u>r-permutation</u> is an ordered arrangement of

Ex:

Ex: If S has n-elements, how many r-permutations does it have?

Note: If n and r are integers with $0 \le r \le n$, then

Combinations:

Def: An r-combination of element of a set is

Ex: How many different teams of 2 students can be formed from

Let $C(n,r)$ denote

Theorem: The number of r-combinations of a set w/ n elements,

Ex: An Ethiopean restaurant has

Sol:

Pf of Thm: Relate r- combinations to r-permutations.

Ex: How many permutations of the letters ABCDEFGH contain

What is relationship between $C(n,r)$ and $C(n,n-r)$?

Lemma:

Pascal's triangle:

Thm: Let n and k be pos. integers w/ $n \geq k$.
Then

Def: A combinatorial proof of an identity

Pf (Combinatorial): Let T be set w/ $n+1$ elements.
LHS above:

§6.4 Binomial Coefficients

The numbers $C(n,r) = \binom{n}{r}$ called binomial coeffs because

Ex: Let's expand out

$$(x+y)(x+y)(x+y)$$

Binomial Theorem: Let x and y be variables and $n \in \mathbb{N}$. Then

$(x+y)^n =$

Pf: We'll give combinatorial proof.

$(x+y)^n =$

Ex: What is coeff. of x^3y^4 in

Cor: Let $n \in \mathbb{Z}_{\geq 0}$ Then

Pf 1:

Pf 2:

Vandermonde's Identity: Let $m, n, r \in \mathbb{N}$, with $r \leq m$, $r \leq n$.

$$\binom{m+n}{r} =$$

Pf (Combinatorial):

Thm: Let $n, k \in \mathbb{N}$, w/ $k \leq n$. Then $\binom{n+1}{k+1} = \sum_{j=k}^{n} \binom{j}{k}$.

Pf (Combinatorial):

More exercises: Prove $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

Prove $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$.