Math 55 Lecture 10

§5.1, §5.2

Induction: Technique for proving

Example of $P(n)$:

To prove $P(n)$ is true for all $n \in \mathbb{Z}^+$, use

Basic step or base case:

Inductive step:

Ex: Prove $P(n)$:

Base case:

Inductive hypothesis.

Want to show $P(k+1)$:

want to show

How can we relate

Ex: Use induction to prove that

$$\sum_{j=0}^{n} ar^j =$$

when $r \neq 1$ and $n \in \mathbb{N}$.

$P(n)$ is proposition

Base case: prove $P(0)$, ie.

Inductive step: show for $k \in \mathbb{N}$,
Inductive hypothesis: Suppose $P(k)$ is true,
ie. Suppose

Want to show

How do we relate
Add

Can also prove inequalities.

Ex: Prove that

$P(n)$ is

Base Case:

Inductive step:

Suppose $P(k)$, i.e.

Want to show $P(k+1)$, i.e.

Can prove divisibility properties.

Ex: Prove that

$P(n)$ is

Base Case: Prove $P(1)$, ie.

Inductive Step: Show $P(k) \Longrightarrow P(k+1)$.

Suppose $P(k)$, ie.

Want to show $P(k+1)$, ie.

How can we relate

Strong Induction for proving $P(n)$, $n \in \mathbb{Z}^+$.

Base Case: Prove $P(1)$.

Inductive Step: Prove that for any $k \in \mathbb{Z}^+$,

If $P(1), P(2), \ldots, P(k)$ are

Ex: Show that if n is an integer greater than 1, then

$P(n)$ is proposition

Proof: Base Case:

Inductive Step:

2 cases:

Note: This completes proof of

Another strong induction example.

Ex: What quantities of money can you make using

Claim: Can form
Let $P(n)$ be

Basis step:

Inductive hypothesis: $P(j)$ is true for

Rk: My basis step consist of 3 statements.
This 3 comes from

Well-ordering property (WOP):

This is an axiom about $\mathbb{Z}^+$ whose truth we assume:
See Appendix 1 for 4 axioms about $\mathbb{Z}^+$:

1.
2.
3.

4.

Can use WOP as tool in proofs.