Math 228B, Spring 2007, Wilkening

Homework 7
due Thursday, May 3

Work in groups of 2-4. Turn in one write-up per group.

Download hw7.zip and unzip it. There are 5 mesh directories:

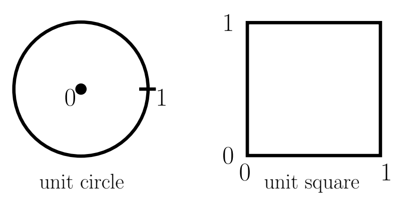
- *circles_lin, circles_quad, circles_iso, squares_lin, squares_quad*

read the notes file in each directory to understand the layout of the data files. See */circles_lin/mesh1.pdf* and */squares_lin/mesh1.pdf* for graphical illustrations. I also included several files containing gaussian quadrature rules of various orders to integrate all polynomials of that order or less *exactly* on the reference triangle.

(1) Solve the Dirichlet problem

$$\begin{aligned} -\Delta u &= 4, && \text{in } \Omega \\ u &= 1 - x^2 - y^2, && \text{on } \partial\Omega \end{aligned}$$

on each of the domains



using linear, quadratic and isoparametric triangular elements; (for the square, there's no need to use isoparametric elements). Compute the H^1 seminorm and L^2 norm of the error exactly for each of the meshes. On the circle, do not neglect the fringe outside the convex hull of the mesh when using linear and quadratic elements. (do neglect it for isoparametric elements). Turn in a table of your errors and log-log plots of the error vs. the mesh parameter h.

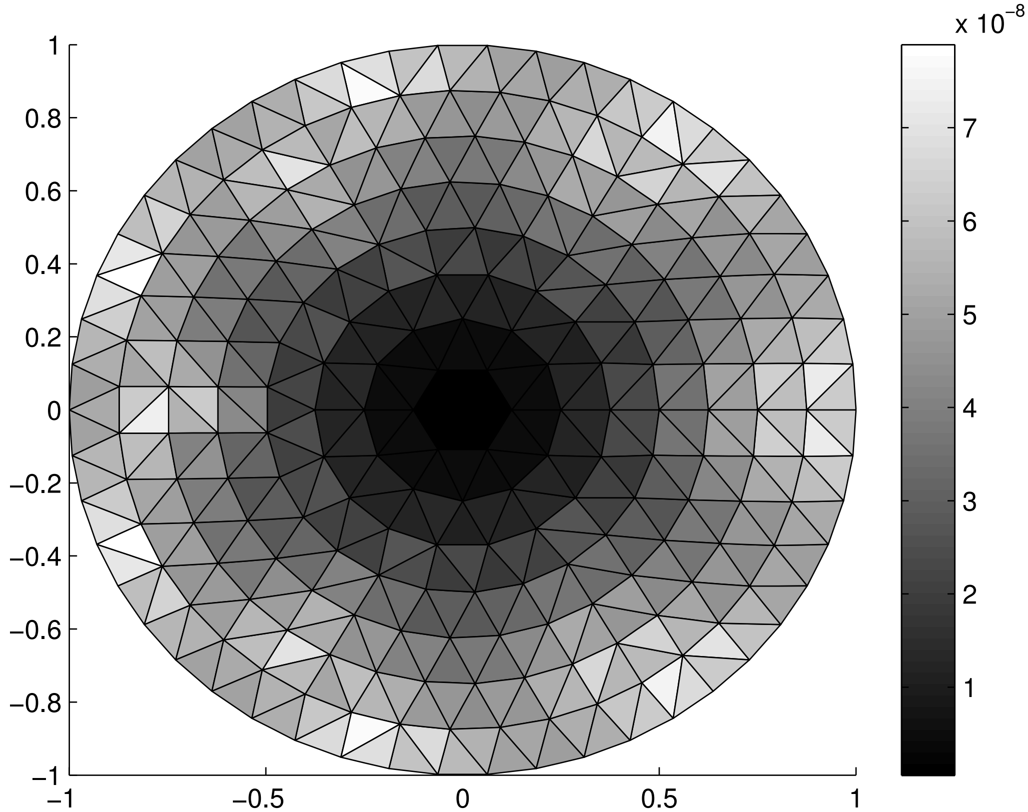
(2) Repeat (1) for the problem

$$\begin{aligned} -\Delta u &= \frac{\pi^2}{4}\left(\cos\frac{\pi r}{2} + \operatorname{sinc}\frac{\pi r}{2}\right), && \text{in } \Omega \\ u &= \cos\frac{\pi r}{2}, && \text{on } \partial\Omega \end{aligned}$$

but this time don't bother with the fringe region outside the convex hull of the mesh $\left(\text{here } r = \sqrt{x^2 + y^2} \text{ and } \operatorname{sinc} z = \frac{\sin z}{z}\right)$. Why do you think the convergence rate for problem (1) is half an order higher than for problem (2) when using isoparametric elements on the circle? To understand what's going on, it may be helpful to plot the contribution to the H^1 and L^2 errors element by element. (see the sample file *plot_errors.m*). For example, in problem 2 with isoparametric elements, I got the following values when I integrated

$$\iint_T \nabla(u_{FE} - u_{exact}) \cdot \nabla(u_{FE} - u_{exact})\, dx\, dy, \qquad (T \in \mathcal{T})$$

over the triangles in *circle_iso/mesh3*:



The H^1 error in the finite element solution is the square root of the sum of all the errors shown here. (I got 0.00400512 for the H^1 error and 7.5355×10^{-5} for the L^2 error on this mesh).