Homework 5 due April 5

(1) Consider the equation $u_t = -u_x$ on the periodic domain $0 \le x \le 10$

with initial conditions $u(x,0) = g(x) = \exp\left[-\frac{1}{2}\min\left((x-1)^2,(x-11)^2\right)\right]\sin(25\pi x)$

(a) plot the FFT of $g(x)$ with $h = \frac{10}{M}$, $M = 512, 1024, 2048$

(plot the magnitudes of the fourier coefficients vs ξ, all on one plot)

(b) compute the group velocity $\gamma(\xi) = \frac{d}{d\xi}\left(\xi\alpha(\xi)\right)$ for {Lax-Wendroff, Leapfrog}

with various choices of ν.

make plots of $\gamma(\xi)$ similar to $\alpha(\xi)$ in the notes.

(c) try out the schemes and solve to $T = 8$ with various choices of ν and M, make plots of things you find interesting and explain what you see

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(2) Change the boundary condition to $u(0,t) = 0$

Change the initial condition to $g(x) = \exp\left(-20\min\left((x-1)^2,(x-11)^2\right)\right)$

try out Lax-Wendroff & Leapfrog treating the b.c. at $x = 10$ via:

(a) impose the illegal condition $u(10,t) = 0$

(b) use upwind for the last point

(c) use a 2nd order scheme for the point u_J that involves u_{J-2}, u_{J-1}, u_J (see Lec 17 for Lax-Wendroff. derive your own for Leapfrog.)

try going to $T = 7, 15, 25, 55$ for starters, but only make plots of interesting results.