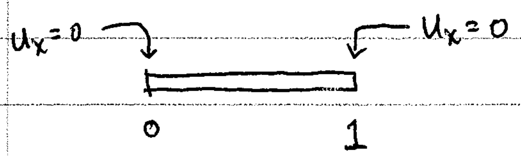
228B Homework 3 due Feb 22

1. Use the Crank-Nicolson scheme to solve the problem:



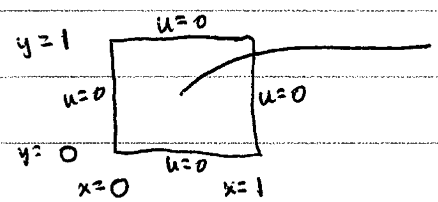
$u_t = u_{xx}$

$u(x,0) = (\sin \pi x)^{100}$

a. plot $u(x,T)$ with $T = 0.1$ and $T = 1$

b. use the refinement path $k = h$ and check the order of convergence of your $T = 0.1$ solution.

2. consider a plate illuminated by a pulsed laser



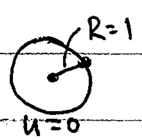
$u_t = u_{xx} + u_{yy} + f(x,y,t)$, $u(x,y,0) = 0$

$f(x,y,t) = 10(\sin \pi x)^{10}(\sin \pi y)^{10}(\sin \pi t)^{10}$

a) use the ADI method to solve for $u(x,y,t)$ to time $t = 1$.

b) plot $u(\frac{1}{2},\frac{1}{2},t)$ for $0 \le t \le 1$ using two choices of k and h that are small enough that the answers look the same

3. consider a hot sphere dropped into a snow bank.



$$\left.\begin{array}{l} u_t = \Delta u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) = u_{rr} + \frac{2}{r}u_r \\ u(r,0) = 1, \quad u_r(0,t) = 0, \quad u(1,t) = 0 \end{array}\right\} \text{final time } T = 0.1$$

plot $u(r,.1)$ for $0 \le r \le 1$

and

plot $u(0,t)$ for $0 \le t \le .1$

a. read §2.16 of the book.

b. implement (2.162a,b) as an explicit scheme $(D_t^+ u_j^n = B u_j^n)$

c. implement (2.162a,b) as an implicit scheme $(D_t^+ u_j^n = B u_j^{n+1})$

d. use the substitution $v(r,t) = r\,u(r,t)$ to simplify the equation and solve for v numerically. How can you recover $u(0,t)$?